October 9: David Vogan (MIT), "Shimura correspondence for spherical representations of split real groups."

(This will be the last forty minutes of my twenty-minute talk at Northeastern, preceded of course by a review of the first twenty minutes. The hard theorems are due to Barbasch, Huang, Adams, Paul, and Trapa.) Suppose $G_{\mathbb{C}}$ is a simply connected complex simple Lie group, and G is a split real form of G. Fix a split torus $T = MA$ of G, and write $W = W(G,T)$ for the Weyl group. There is a unique connected double cover $\widetilde{G}$ of G, with split torus $\widetilde{T} = \widetilde{M}A$. The (usually non-abelian) finite group $\widetilde{M}$ has certain distinguished representations $\widetilde{\delta}$; we fix one of these. The principal series of $\widetilde{G}$ attached to $\widetilde{\delta}$ behaves formally much like the spherical principal series of G. In particular, it is natural to ask for a description of the complementary series

$$C(\widetilde{G},\widetilde{\delta}) = \{\nu \in \mathfrak{a}_0^* \mid J(\widetilde{\delta},\nu) \text{ is unitary}\}.$$

For each root of T in G, we get a root $SL(2,\mathbb{R})$. The covering $\widetilde{G}$ pulls back to a double cover of $SL(2,\mathbb{R})$. We can define now a new root system in $\mathfrak{a}_0^*$, by dividing the old root by two (if this covering is non-trivial) or leaving the old root unchanged (if the covering is trivial). Let H be a split simple group with this new root system. The main conjecture is that the complementary series for $\widetilde{G}$ and $\widetilde{\delta}$ is identical to the spherical complementary series for H:

$$C(\widetilde{G},\widetilde{\delta}) = C(H,1) \subset \mathfrak{a}_0^*.$$

(This equality is the "Shimura correspondence" of the title.) I'll explain how work of Barbasch offers an approach to the conjecture, and proves the inclusion $\subset$ for G classical.