October 23: Michael Shapiro (Michigan State), "Cluster algebras and Poisson geometry"

A cluster algebra of rank n is a commutative ring generated inside its ambient field by a certain distinguished family of generators called cluster variables. These generators are obtained from some initial cluster by an explicit "mutation" process. The initial study of cluster algebras started by A.Zelevinsky and S.Fomin is motivated by dual canonical bases and total positivity in semisimple groups. Coordinate rings of such varieties, as, for instance, Bruhat cells in flag varieties and Grassmanians possess cluster algebra structures.

Poisson bracket on a manifold is a skew-commutative bilinear operation on the space of functions of this manifold, that turns the space of functions into Lie algebra. Symplectic structure on the manifold is a nondegenerate closed differential 2-form on this manifold. There is a natural correspondence between symplectic structures and nondegenerate Poisson bracket.

We introduce a notion of Poisson bracket and closed differential 2-form on R^n compatible with the cluster algebra structure and describe all such Poisson brackets and 2-forms. Moreover, any cluster algebra gives rise to a unique symplectic form (equivalently, nondegenerate Poisson bracket) on some affine space, generally speaking, of smaller dimension.

We hope that these objects shed some light on cluster algebra axioms.