October 30: Pavle Pavle Pandžić (University of Zagreb), "Dirac cohomology and $\mathfrak{u}$ cohomology." FOLLOWED BY DINNER.

Following the recent results of Huang-Pandžić and Kostant on Dirac cohomology, Vogan has suggested to study the case of a θ-stable parabolic $\mathfrak{q} = \mathfrak{l} \oplus \mathfrak{u}$ and examine the relationship between the Dirac cohomology of a $(\mathfrak{g}, K)$-module X for the Kostant's cubic Dirac operator D corresponding to $\mathfrak{l}$, $\overline{\mathfrak{u}}$-cohomology of X and $\mathfrak{u}$-homology of X. Namely, all of these can be defined in terms of the module $X \otimes S$ where S is a space of spinors for $\mathfrak{u} \oplus \overline{\mathfrak{u}}$, and D is roughly the sum of the two (co)boundary operators.

In a joint work with J.-S. Huang and D. Renard, we interpret this equality in the algebra $U(\mathfrak{g}) \otimes C(\mathfrak{s})$ and show a direct relationship between Casselman-Osborne theorem and Vogan's conjecture proved by Huang-Pandžić and Kostant.

In a special case, for a Hermitian symmetric pair (G, K) and $L = K$, we get that the three cohomology spaces are equal up to a character of L if X is unitarizable. For general X, this is not true already for $sl(2)$. An explanation why it fails is that the homological properties of Dirac cohomology are different from those of Lie algebra cohomology. In particular, a short exact sequence of $(\mathfrak{g}, K)$-modules induces a six-term exact sequence of Dirac cohomologies.