

MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
DEPARTMENT OF MATHEMATICS

# Geometric Analysis Seminar

Wednesday, October 13, 2021

4:00pm – 5:00pm      Room 2-131

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## “The Yau-Tian-Donaldson conjecture for general polarized manifolds”

### Abstract

According to the Yau-Tian-Donaldson conjecture, the existence of a constant scalar curvature Kähler (cscK) metric in the cohomology class of an ample line bundle  $L$  on a compact complex manifold  $X$  should be equivalent to an algebro-geometric “stability condition” satisfied by the pair  $(X, L)$ . The cscK metrics are the critical points of Mabuchi’s K-energy functional  $M$ , defined on the space of Kähler potentials, and an important result of Chen-Cheng shows that cscK metrics exist iff  $M$  satisfies a standard growth condition (coercivity/properness). Recently the speaker has shown that the K-energy is indeed proper if and only if the polarized manifold is stable. The stability condition is closely related to the classical notion of Hilbert-Mumford stability. The speaker will give a non-technical account of the many areas of mathematics that are involved in the proof. In particular, he hopes to discuss the surprising role played by arithmetic geometry in the spirit of Arakelov, Faltings, and Bismut-Gillet-Soule.