COMBINATORICS SEMINAR Convex Geometry of Orbits

Greg Blekherman University of Michigan Ann Arbor

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Abstract:

The talk will focus on the study of metric properties of convex bodies B and their polars B^o , where B is the convex hull of an orbit under the action of a compact group G. Examples include the Traveling Salesman Polytope in polyhedral combinatorics $(G = S_n, \text{ the symmetric group})$, the set of non-negative polynomials in real algebraic geometry (G = SO(n), the special orthogonal group), and the convex hull of the Grassmannian and the unit comass ball in the theory of calibrated geometries (G = SO(n), but with a different action). We will discuss several results that allow us to conclude that there are substantially more nonnegative polynomials than sums of squares, prove existence of "deep" faces of the real Grassmanian and compute the radius of the largest ball contained in the symmetric Traveling Salesman Polytope. All of the above results use the same unified framework. Our main tool is a new simple description of the ellipsoid of largest volume contained in B^o . This is joint work with Sasha Barvinok.