COMBINATORICS SEMINAR
Zero Distributions For Polynomials Arising In Combinatorics

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December 14, 2005

Abstract:

Many natural sequences of polynomials especially those from combinatorics have interesting limiting sets of zeros in the complex plane as their degrees go to infinity. R. Stanley complied a list of many interesting examples. In this talk, I will discuss joint work with William M.Y. Goh in investigating several of his problems including Euler and Bernoulli polynomials and the partition polynomials $F_n(x)$ given as $\sum_{k=0}^{n} p_k(n)x^k$ where $p_k(n)$ is the number of partitions of n with exactly k parts. I will try to give a flavor of the analysis and the role of computation in our work.

The limiting behavior for the Euler and Bernoulli polynomials is related to the Szegö curve which governs the zeros for the taylor polynomials of the exponential.

In Stanley's plot for the partition polynomial of degree 200, its zeros cluster around the unit circle together with a sparse family inside the unit disc. Initially it was unclear what role such zeros play in the limit.

Using techniques from analytic number theory and extensive numerical computation to degree 30,000, we found that the zeros approach the unit circle as well as three curves inside the disc given implicitly in terms of $f_k(z) = \Re[\sqrt{\operatorname{Li}_2(z^k)}]/k$ where Li_2 is the dilogarithm. We have reduced the proof to a conjecture about domination among the functions f_k.