Abstracts

PuMa GraSS

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Multiplicity Problems in Geometric Analysis

Ao Sun

In many geometry problems, the convergence of a sequence of geometric objects may have multiplicity. When the multiplicity is greater than 1, we will lose much information under the convergence, such as the topology, the curvature, etc. Thus it is important to avoid higher multiplicity or at least try to understand what happens when higher multiplicity appears. In this talk, I will introduce the precise meaning of multiplicity, and discuss some famous problems which are related to the multiplicity.

Tight Contact Structures on a Solid Torus

Zhenkun Li

The classification of tight contact structures on a give 3-manifold is a fundamental question in the study of 3-dimensional contact geometry. However, only a few cases have been understood and among them, solid torus is a simple but important example. In this talk I will focus on the classification of tight contact structures on a solid torus, a result originally due to Honda (2000). I will start with the basic definitions and tools such as tightness, convexity, by-passes, etc. I will also introduce how Floer homologies play a role in detecting and distinguishing tight contact structures.

Mixed Volumes

Alexey Balitskiy

The notion of mixed volume is the polarization of the usual volume of a convex body. One of the most obscure theorems of classical convexity is the Aleksandrov–Fenchel inequality on mixed volumes, for which no purely geometric proof is known (I'm exaggerating, but not too much). I'll sketch some of the approaches to prove it, and derive a few Brunn–Minkowski-like and isoperimetric-like corollaries of that remarkable theorem.

Brownian motion, Lie algebras, and a surprising duality

Roger Van Peski

I'll give an informal introduction to what Brownian motion is (have no fear, the word sigma-algebra shall be said exactly zero times), then start exploring how it behaves differently in different dimensions and drawing pictures. It turns out that there is a very surprising duality between Brownian motions in dimensions 1 and 3, of which one direction tells us roughly that a 1-dimensional Brownian motion conditioned to stay in the positive real numbers behaves exactly the same as the norm of a vector of 3 Brownian motions. What's even more amazing is that this duality is the simplest nontrivial case of a general phenomenon regarding Brownian motion on Lie algebras, which in turn is a 'semiclassical limit' of discrete Markov chains on their representations. If there's extra time I'll talk briefly about how this comes up in disguise in random matrix theory.

Invitation to sympletic geometry

Tim Large

The Method of Chabauty and Coleman

Nicholas Triantafillou

Given a curve C of genus $g \geq 2$ over a number field K, Faltings' Theorem says that the set $C(K)$ is finite. But Faltings' Theorem is ineffective - it doesn't tell you how to compute the set $C(K)$. We will introduce (Coleman's version of) Chabauty's method, and use it to compute $C(K)$ for some specific curves.

The Four-color theorem and Gauge theory

Mariano Echeverria

While it is relatively straightforward to understand the statement of the Four-Color Theorem, one of its interesting features is that the original proof given by Appel and Haken (as well as the more modern proofs) rely to some extent on the use of computers. Hence it has been of interest to mathematicians whether or not it is possible to find a computer-free proof.

I will explain one candidate for such a proof, as described by Kronheimer and Mrowka in recent years. An interesting feature of their proposal is that it uses many powerful techniques from 3-manifold topology, including Gabai's sutured manifold theory and a version of gauge theory (namely, Instanton Floer Homology).

In particular, I will explain how to associate to a trivalent graph G (embedded in 3-space) a finite dimensional vector space J(G) and how $J(G)$ is related to the original 4-color theorem.

An Overview of the Sato-Tate Conjecture

Vishal Arul

Given a system of polynomial equations, what can one say about the distribution of the number of solutions modulo p? We will first warm up with the zero dimensional case where we have one polynomial in one variable; the answer here is given by the Chebotarev density theorem. Next, we will consider the case of an elliptic curve; the Sato-Tate conjecture gives a prediction here (which has been proven for elliptic curves defined over $\mathbb{Q}$). We will state the Sato-Tate conjecture and provide examples of elliptic curves and their predicted Sato-Tate distributions, along with data from computer experiments that agree with these predictions.

Understanding 2×2 matrix multiplication

Sveta Makarova

To start, I will formulate a classical fact that one can multiply two 2×2 matrices using just 7 multiplications instead of the expected 8. Then I will explain why this means that a certain cubic tensor lies on the variety of secant 6-spaces to a Segre variety. This geometric problem turns out to be unexpectedly interesting, and to demonstrate the nontriviality, I will switch to discussing a simpler case of $2 \times 2 \times 2$ tensors.

A Taste of non-asymptotic random matrix theory

Vishesh Jain

Classical random matrix theory is mostly concerned with the asymptotic spectral properties of random matrices as their dimensions go to infinity. In contrast, non-asymptotic random matrix theory seeks to study such questions in fixed dimensions. I will provide a very informal introduction to some of the ideas in this area via Komlos's classical upper bound on the probability that a random $n \times n$ Bernoulli matrix is singular.

Naive introduction to Minimal Model Program

Kai Huang

The minimal model program (MMP) aims to construct a birational model of any projective variety which is as simple as possible. The subject has its origins in the classical birational geometry of surfaces studied by the Italian school, and the 3-dimensional case was completed (at least over complex numbers) by Mori in 1980s. I will give an informal introduction to the basic idea of MMP, and then talk about the surface case. I will also talk about the application of MMP, especially in the study of Kähler–Einstein metric.