Onboarding Instructors in an Active Linear Algebra Course

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Dedication

Alfonso Gracia-Saz
1976–2021
Educator, Mentor, Friend

Post your memories of Alfonso at
https://uoft.me/alfonso-memorial
The Course

Linear Algebra I, MAT223 (*per semester*)
- ~1100 students
- ~7 sections
- ~7 instructors (mostly inexperienced)
- First-year students (mostly inexperienced)
Redesign Linear Algebra I, MAT223

- Revamp curriculum
- Provide a uniform student experience
- Introduce active-learning teaching strategies in every section
The Project (c. 2018)

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The Principal Strategy

What to do?

Provide training and support.

Lower the barrier-of-entry to active learning.

Provide high-quality materials and guides.
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- Lower the barrier-of-entry to active learning.
  - Provide high-quality materials and guides.
What does active learning look like in MAT223?

27.1 Draw \( \vec{u} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \) and all vectors orthogonal to it. Call this set \( A \).

27.2 If \( \vec{x} = \begin{bmatrix} x \\ y \end{bmatrix} \) and \( \vec{x} \) is orthogonal to \( \vec{u} \), what is \( \vec{x} \cdot \vec{u} \)?

27.3 Expand the dot product \( \vec{u} \cdot \vec{x} \) to get an equation for \( A \).

27.4 If possible, express \( A \) as a span.
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1. Set the stage

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2. Individual think time

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3. Small group discussion
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3. Small group discussion
4. Full class discussion
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5. Wrap-up

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**What are some prerequisites/challenges in teaching this way?**

Share your thoughts at:

https://tinyurl.com/ACTIVE-ONBAORDING
My Journey (Year 1)

Goal: Provide materials covering every day of class.
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Year 1: The Materials
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- Assembled together a coherent set of exercises for the entire semester (aligning with content)
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- Assembled together a coherent set of exercises for the entire semester (aligning with content)
- Created instructor guide to in-class exercises.
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Example instructor margin notes:

**Generate lines using orthogonality.**
The goal of this problem is to
- Visually see how the set of all vectors orthogonal to a given vector forms a line.
- Given a line defined as the set of all vectors orthogonal to a given vector, express the line using an equation or span.

**Notes/Misconceptions**
- This problem won’t be hard, so don’t spend too much time on it.
- For part 1, students might insist on drawing arrowheads and tails on their vectors. This is an opportunity to discuss when you should draw arrowheads/tails and when you shouldn’t.
Example instructor margin notes:

**Year 1: Instructors**

- Provided with in-class exercises & instructor guide.
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- Provided with in-class exercises & instructor guide.
- Asked to do “active learning” at least 3 days in the semester.
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- Asked to do “active learning” at least 3 days in the semester.

Support provided:

- Open invitations to visit my lecture.
- Teaching observations.
My Journey (Year 1)

Year 1 Outcomes

1/3 of instructors used materials every day. Everyone tried on the 3 special days. What I saw in the classroom varied from ☺ to 😭😢.

Challenges
Instructors felt uncomfortable “ordering students around” (e.g., forcing discussion, etc.)

One instructor “gave up” and lectured directly from the book.

2/3 of instructors didn’t adopt an active-learning style.
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What did I do?

- Salesmanship
- Training
- Wrote a textbook
- Pre-reading/pre-meeting on active learning
- Visit my lecture (before semester starts; mandatory)
- Pre-term coaching
- Weekly meetings
- Classroom observations
- Textbook integrates with in-class exercises
- Doubles as workbook for students (they need a physical copy to write in)
My Journey (Year 2)

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Now And the Future

The Current Situation

Everything is butterflies and rainbows…

All instructors use the in-class exercises

Students like the textbook/workbook

Instructor prep time decreased

When I visited the classroom…

Instructors were trying

Most were really bad, with the same problems as during pre-term coaching

It’s as if four hours of training are not enough!
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Step 1:

Class Observation & Pre-meeting
Instructors read a short article (the Feeman et. al. paper) and observe my classroom. Pre-meeting to discuss logistics; Active Learning is just the way the class works.

Step 2:
Coaching through Role-playing
Instructors prepare lessons 1 & 2. Outline of "four stages": Introduce problem, Facilitate student work, Whole-class discussion, Wrap-up. Instructors try, are interrupted, and retry.
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Weekly meetings

Ask about class

Focus on content-specific goals (room for improvement; could include teaching goals!)

Step 3:

Classroom observations

Time consuming, but valued by all

Twice a term (energy permitting)

Step **:

TA Support

Lecture TAs for every day of class

Lecture TAs are trained in both how to help students and how to help instructors.
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Thank You
Instructor Difficulties

Role Playing *Common Issues*

Instructors tend to...
- Give hints on a problem before students start.
- Explain details without measuring student’s knowledge.
- Offer their own explanations rather than build off student knowledge.

All of these issues come up and are discussed.

Classroom *Common Issues*

- Everything from role playing, and
- Suggestions rather than instructions (e.g., “if you’d like, discuss with your group”).
- Having “office hour”-style discussions instead of whole-class discussions.
- Transitions, transitions, transitions!