MAT134H5 - Integral Calculus for Life Sciences

Guidelines for Section 6.1: Volumes Using Cross Sections

In this section we learn how to use definite integrals to compute volumes of three-dimensional solids.

Recall that the definition of the definite integral was originally motivated by problem of **finding areas** of plane regions. It is quite remarkable that definite integrals can also be used to compute volumes, and there is no need to develop a new tool for this purpose.

Throughout the section you will see many three-dimensional diagrams. Use them to gain some intuition and become comfortable in working with 3D objects. This will help you set up properly the integrals for finding volumes of various 3D solids.

Suggested Video(s)

- `https://www.youtube.com/watch?v=KgSTjyA-3-Y`

Reading Instructions

- Read pages 368-370 carefully. The formula $V = \int_a^b A(x)\,dx$ is the most important formula in this section. In fact, there is no need to memorize any of the other formulas, as they all follow from this general result. Read Examples 1-3, and make an effort to understand the connection between the diagrams and the computations.

- The remaining of the section focuses on **Solids of Revolution**, solids obtained by revolving a plane region around a fixed axis.

- Read Examples 4-10. Can you see when we should use disks and when washers?

- In Example 7, 8 and 10, the integrals are set up in terms of the y variable. This approach is more natural and convenient when dealing with regions that revolve around a **vertical axis**.

  These examples may seem confusing, but in fact, the approach and techniques used here are the same as the ones used in the other examples. The only difference is that we use y as our integration variable (instead of x).