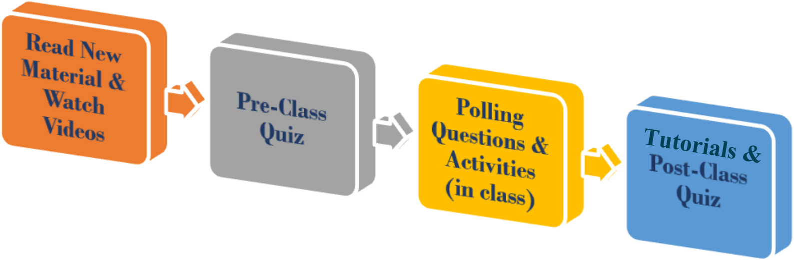
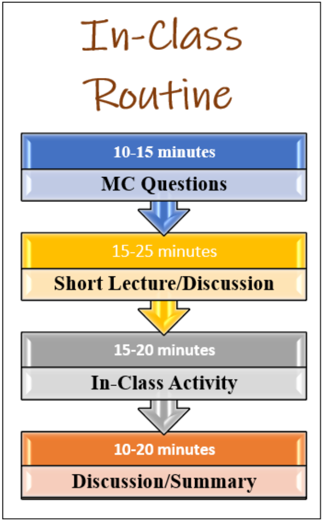
A Flipped Classroom Demonstration

Shay Fuchs

University of Toronto Mississauga

Course Structure




In-Class Routine
10-15 minutes
MC Questions
15-25 minutes
Short Lecture/Discussion
15-20 minutes
In-Class Activity
10-20 minutes
Discussion/Summary

Section 6.1

Volumes Using Cross Sections

Get Ready to Vote!

Volumes Using Cross Sections (6.1)

The following formula is used to compute volumes

of solids using cross sections: $\mathbf{V} = \int_{\mathbf{a}}^{\mathbf{b}} \mathbf{A(x)}\,\mathbf{dx}$.

What does the function $\mathbf{A(x)}$ represent?

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(A) The **surface area** of the solid.

(B) The **area of a cross section** at $\mathbf{x}$.

(C) The **volume of the solid** on the interval $[\mathbf{a}, \mathbf{x}]$.

(D) The **volume of the solid** on the interval $[\mathbf{x}, \mathbf{b}]$.

(E) The **area of the base** of the solid.

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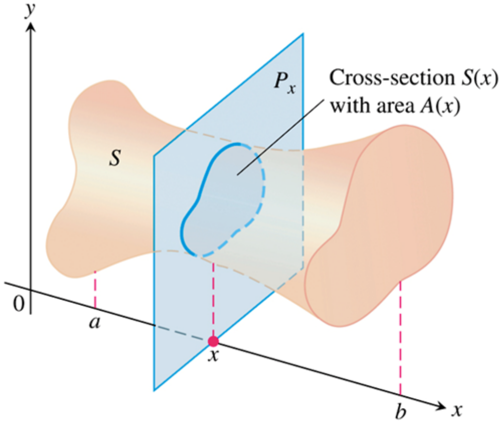
Answer: **(B)** .

Volumes Using Cross Sections (6.1)

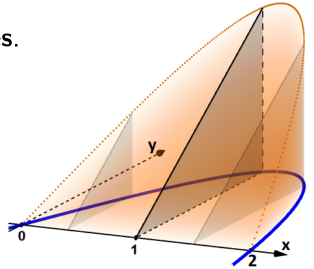


Volumes Using Cross Sections (6.1)

The diagram shows a solid, whose base is the region between the curve $\mathbf{y = 4x - x^3}$ and the x-axis, for $\mathbf{0 \leq x \leq 2}$.

Its cross sections are **isosceles right triangles**.

What is the function $\mathbf{A(x)}$ in this case?



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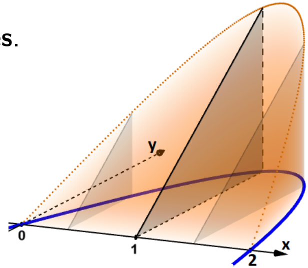

(A) $\frac{1}{2}(4x - x^3)^2$

(B) $(4x - x^3)^2$

(C) $4x - x^3$

(D) $x \cdot (4x - x^3)$

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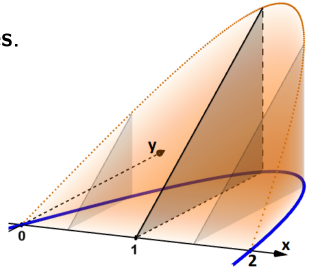

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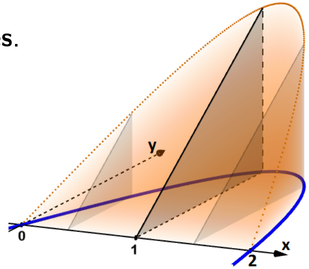

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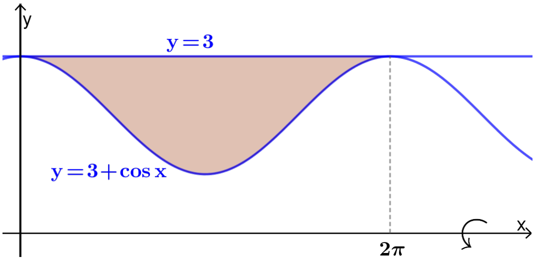
Answer: (A), as each leg of a cross sectional triangle at x has length $4x - x^3$.

Volumes Using Cross Sections (6.1)

The shaded region rotates around the x-axis.

What is **the shape of a typical cross section**?



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(A) A square.

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(D) A disk.

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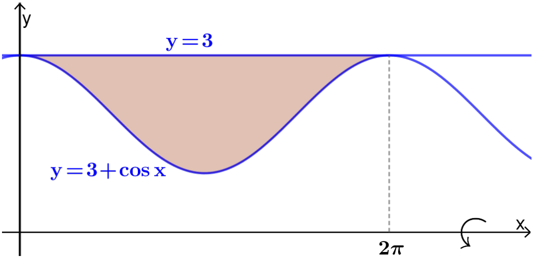

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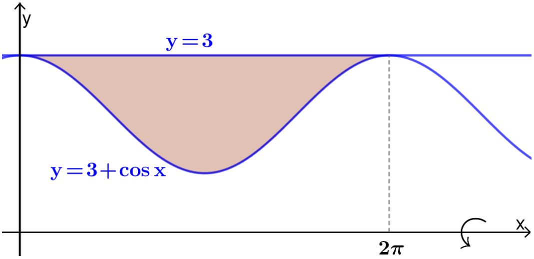

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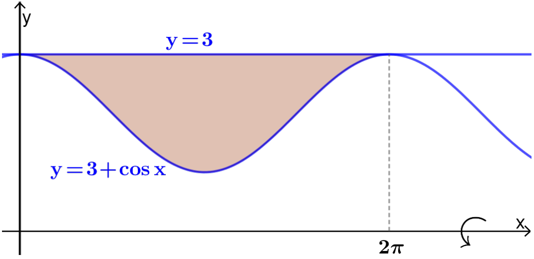

Answer: **(E)** .

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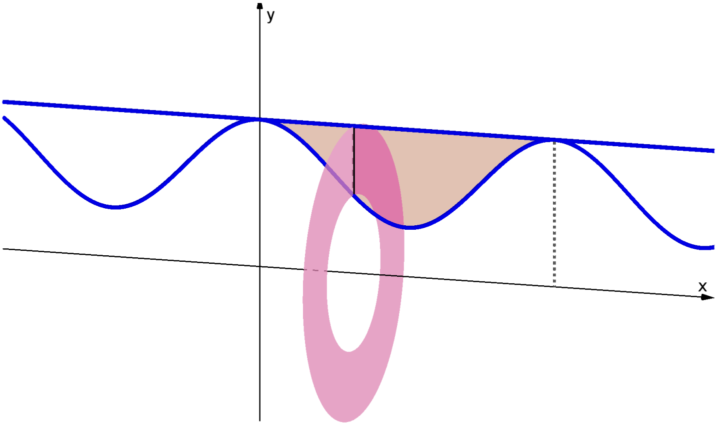
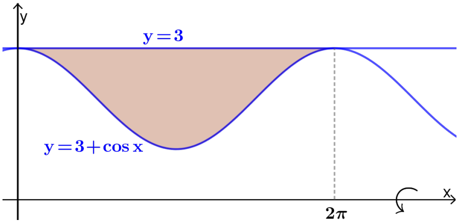


Volumes Using Cross Sections (6.1)

Which integral gives **the volume** of the resulting solid? ? ? ?



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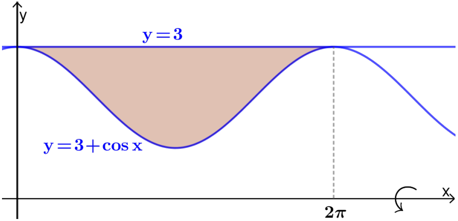

(A) $\pi \int_0^{2\pi} [3 - (3 + \cos x)]^2 \, dx$

(B) $\pi \int_0^{2\pi} \left[3^2 - (3 + \cos x)^2\right] dx$

(C) $\pi \int_0^{2\pi} (3 + \cos x)^2 dx$

(D) $\pi \int_0^{2\pi} \cos^2 x \, dx$

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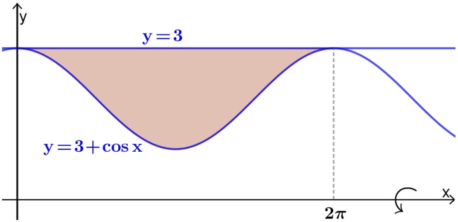

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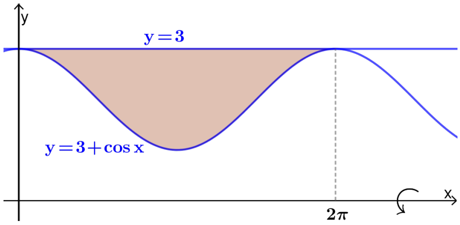

(C) $\pi\int_0^{2\pi}(3+\cos x)^2 dx$

(D) $\pi\int_0^{2\pi}\cos^2 x\,dx$

(E) $\pi\int_0^{2\pi}(-\cos x)\,dx$

Answer: (B) , as the volume is given by $V=\pi\int_a^b\left[R^2(x)-r^2(x)\right]dx$.

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- If the cross-sections are **disks**, then $\mathbf{A(x)} = \pi[\mathbf{R(x)}]^2$, and

$$\mathbf{V} = \pi \int_a^b [\mathbf{R(x)}]^2 \, \mathbf{dx} \ .$$

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- If the cross-sections are **washers**, then $\mathbf{A(x)} = \pi[\mathbf{R(x)}]^2 - \pi[\mathbf{r(x)}]^2$, and

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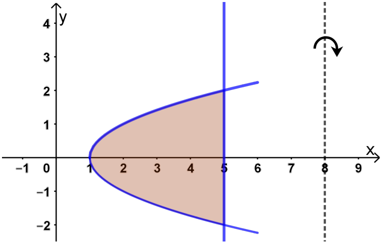
- When we rotate a region around a **vertical axis**, we need to integrate with respect to $\mathbf{y}$.

Volumes Using Cross Sections (6.1)

Find the volume of the solid obtained by revolving the region bounded by the curves $x = y^2 + 1$ and $x = 5$, around the line $x = 8$.



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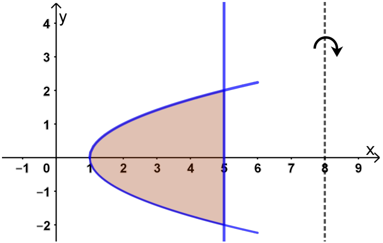

Guidelines:

- Integrate with respect to y.
- What is the shape of the cross sections?

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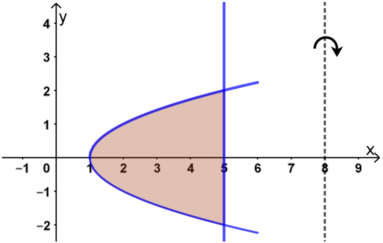

Guidelines:

- Integrate with respect to y.
- What is the shape of the cross sections?
- Find **the inner** and **the outer radius** (as a function of y).
- Set up the integral and compute it.