A Flipped Classroom Demonstration

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Course Structure

1. Read New Material & Watch Videos
2. Pre-Class Quiz
3. Polling Questions & Activities (in class)
4. Tutorials & Post-Class Quiz
In-Class Routine

10-15 minutes
MC Questions

15-25 minutes
Short Lecture/Discussion

15-20 minutes
In-Class Activity

10-20 minutes
Discussion/Summary
Section 6.1

Volumes Using Cross Sections

Get Ready to Vote!
The following formula is used to compute volumes of solids using cross sections:

\[ V = \int_{a}^{b} A(x) \, dx. \]

What does the function \( A(x) \) represent?

(A) The surface area of the solid.
(B) The area of a cross section at \( x \).
(C) The volume of the solid on the interval \([a, x]\).
(D) The volume of the solid on the interval \([x, b]\).
(E) The area of the base of the solid.

Answer: (B).
Volumes Using Cross Sections (6.1)

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Answer: (B).
Volumes Using Cross Sections (6.1)

The diagram shows a solid, whose base is the region between the curve \( y = 4x - x^3 \) and the x-axis, for \( 0 \leq x \leq 2 \).

Its cross sections are isosceles right triangles.

What is the function \( A(x) \) in this case?

Answer: (A), as each leg of a cross sectional triangle at \( x \) has length \( 4x - x^3 \).
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Its cross sections are **isosceles right triangles**.

What is the function \( A(x) \) in this case?

(A) \( \frac{1}{2} (4x - x^3)^2 \)

(B) \( (4x - x^3)^2 \)

(C) \( 4x - x^3 \)

(D) \( x \cdot (4x - x^3) \)

(E) \( \frac{1}{2} (4x - x^3) \)

Answer: (A), as each leg of a cross sectional triangle at \( x \) has length \( 4x - x^3 \).
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- (E) \(\frac{1}{2}(4x - x^3)\)

**Answer:** (A), as each leg of a cross sectional triangle at \(x\) has length \(4x - x^3\).
Volumes Using Cross Sections (6.1)

The shaded region rotates around the x-axis.

What is the shape of a typical cross section?

Answer: (E)
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What is the shape of a typical cross section?

(A) A square.
(B) A triangle.
(C) A parabola.
(D) A disk.
(E) A washer.

Answer: (E)
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Answer: (E)
Volumes Using Cross Sections (6.1)
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Which integral gives the volume of the resulting solid?

\[
\pi \int_0^2 \left[ 3 - (3 + \cos x)^2 \right] dx
\]

Answer: (B), as the volume is given by

\[
V = \pi \int_a^b \left[ R^2(x) - r^2(x) \right] dx
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Volumes Using Cross Sections (6.1)

Which integral gives the volume of the resulting solid?

(A) \( \pi \int_{0}^{2\pi} [3 - (3 + \cos x)]^2 \, dx \)

(B) \( \pi \int_{0}^{2\pi} [3^2 - (3 + \cos x)^2] \, dx \)

(C) \( \pi \int_{0}^{2\pi} (3 + \cos x)^2 \, dx \)

(D) \( \pi \int_{0}^{2\pi} \cos^2 x \, dx \)

(E) \( \pi \int_{0}^{2\pi} (-\cos x) \, dx \)

Answer: (B), as the volume is given by \( V = \pi \int_{a}^{b} [R(x)^2 - r(x)^2] \, dx \).
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Answer: (B), as the volume is given by \[ V = \pi \int_{a}^{b} [R^2(x) - r^2(x)] \, dx \].
Volumes Using Cross Sections (6.1)

Highlights:

- **Main Formula**: \( V = \int_a^b A(x) \, dx \).
  - If the cross-sections are disks, then \( A(x) = \pi \left[ R(x)^2 \right] \), and \( V = \pi \int_a^b \left[ R(x)^2 \right] \, dx \).
  - If the cross-sections are washers, then \( A(x) = \pi \left[ R(x)^2 - r(x)^2 \right] \), and \( V = \pi \int_a^b \left[ R(x)^2 - r(x)^2 \right] \, dx \).
  - When we rotate a region around a vertical axis, we need to integrate with respect to \( y \).
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V = \pi \int_{a}^{b} \left( \left[ R(x) \right]^2 - \left[ r(x) \right]^2 \right) dx.
\]

• When we rotate a region around a **vertical axis**, we need to integrate with respect to \( y \).
Volumes Using Cross Sections (6.1)

Find the volume of the solid obtained by revolving the region bounded by the curves \( x = y^2 + 1 \) and \( x = 5 \), around the line \( x = 8 \).
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**Guidelines:**

- Integrate with respect to \( y \).
- What is the shape of the cross sections?
Volumes Using Cross Sections (6.1)

Find the volume of the solid obtained by revolving the region bounded by the curves $x = y^2 + 1$ and $x = 5$, around the line $x = 8$.

Guidelines:

- Integrate with respect to $y$.
- What is the shape of the cross sections?
- Find the inner and the outer radius (as a function of $y$).
- Set up the integral and compute it.