Optimization I

Instructions
There are two problems. Complete as many problems as time allows, recording your final answers. The group member with the first name in alphabetical order will be the group scribe today. Note: These problems are adapted from Thomas’ Calculus (Weir, 2005).

Section A: Main Material
(Manufacturing) Suppose that \( c(x) = x^2 - 20x + 20,000 \) is the cost of manufacturing \( x \) items.
   a. Find a production level that will minimize the average cost per item of making \( x \) items.
   b. After finding your answer above, plot the average cost function and confirm your answer graphically.
   c. Say that the company is constrained to make at least 15 items. Find a production level that would minimize the average cost per item in that case.

Section B: Enrichment and Challenge Material
Prove that the production level (if any) at which average cost is smallest is a level at which the average cost equals marginal cost. (Marginal cost is the rate of change of cost with respect to production level.)

Citations