APPLIED MATHEMATICS COLLOQUIUM

Vector Diffusion Maps and the Connection Laplacian

Amit Singer
(Princeton University)

Abstract: Motivated by problems in structural biology, specifically cryo-electron microscopy, we introduce vector diffusion maps (VDM), a new mathematical framework for organizing and analyzing high dimensional data sets, 2D images and 3D shapes. We introduce the discrete graph connection Laplacian which is a generalization of the graph Laplacian from spectral graph theory. The connection Laplacian provides an extension of diffusion maps and other non-linear dimensionality reduction methods, such as LLE, ISOMAP and Laplacian eigenmaps. While existing methods are either directly or indirectly related to the heat kernel for functions over the data, VDM is based on the heat kernel for vector fields. VDM provides tools for organizing complex data sets, embedding them in a low dimensional space and interpolating and regressing vector fields over the data. In particular, it equips the data with a metric, which we refer to as the vector diffusion distance. In the manifold learning setup, where the data set is distributed on a low dimensional manifold M^d embedded in R^p, we prove the relationship between VDM and the connection-Laplacian operator for vector fields over the manifold. Applications to structural biology (cryo-electron microscopy and NMR spectroscopy), computer vision and shape space analysis will be discussed. (Joint work with Hau-tieng Wu and others that will be mentioned in the talk.)

Monday April 30, 2012
4:30 PM
Building 2, Room 105
Refreshments are available in Building 2, Room 290
(Math Common Room) between 3:30 – 4:30 PM

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Massachusetts Institute of Technology
Department of Mathematics
Cambridge, MA 02139