

APPLIED MATHEMATICS COLLOQUIUM

GENERATING FUNCTION ALGORITHMS AND SOFTWARE FOR INTEGER OPTIMIZATION

JESUS DE LOERA

University of California, Davis

ABSTRACT:

In the 1990s V.A. Barvinok proposed rational generating functions as the ideal fast data structure for representing or encoding lattice points inside polyhedra. In this talk, we explain how rational functions (of various types) have found interesting use in the theory and practice of integer optimization. In particular, we discuss the following theorem that generalizes H. Lenstra Jr.'s classical result on linear integer programming in fixed dimension:

Theorem: Let the number of variables d be fixed. Let $f(x_1, \dots, x_d)$ be a polynomial of maximum total degree D with integer coefficients, and let P be a convex rational polytope defined by linear inequalities in d variables. We obtain an increasing sequence of lower bounds $\{L_k\}$ and a decreasing sequence of upper bounds $\{U_k\}$ to the optimal value

$$f^* = \text{maximize } f(x_1, x_2, \dots, x_d) \text{ subject to } x \in P \cap \mathbb{Z}^d. \quad (1)$$

The bounds L_k, U_k can be computed in time polynomial in k , the input size of P and f , and the maximum total degree D , and they satisfy the inequality $U_k - L_k \leq f^* \cdot (\sqrt[k]{|P \cap \mathbb{Z}^d|} - 1)$.

More strongly, if f is non-negative over the polytope (i.e. $f(x) \geq 0$ for all $x \in P$), there exists a fully polynomial-time approximation scheme (FPTAS) for the optimization problem.

I will also discuss the practical side of these algorithms. For example, the software *LattE*, developed by our group, has solved some very nasty knapsack problems within minutes of computation. I will conclude my talk with some fascinating connections to pure combinatorics.

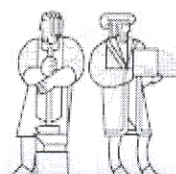
MONDAY, DECEMBER 18, 2006

4:30 PM

Building 2, Room 105

*Refreshments at 4:00 PM in Building 4, Room 174
(Math Majors Lounge)*

Applied Math Colloquium: <http://www-math.mit.edu/amc/fall06>
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Massachusetts Institute of Technology
Department of Mathematics
Cambridge, MA 02139