Lucy Cai

Optimal Heating on Parallelogram Tori

under the direction of Catherine Cawley Wolfram

Abstract

If N heat sources are placed onto a closed torus and taken away at time $t = 0$, surface temperature will initially spike at these N points and eventually converge to some equilibrium. An interesting question to consider is, what is the optimal placement of heat sources so that surface temperature reaches equilibrium at the quickest rate? Solutions to this question are known for square tori S. We instead explore solutions for various parallelogram tori, represented by lattice grids on which the fundamental domain is a parallelogram $P = A(S)$ for $A \in GL(2, \mathbb{Z})$. In particular, we use Fourier series to show that if $\{(z_n, w_n)_{n=1}^N\}$ is an optimal solution on S, then $\{A(z_n, w_n)_{n=1}^N\}$ is an optimal solution on P, where decay rate is preserved up to a change in constant.

Yunseo Choi

On Two-sided Matching in Infinite Markets

under the direction of Prof. Scott Duke Kominers

Abstract

In this paper, we lift a number of classic results for one-to-one matching markets, such as group strategy proofness, comparative statics, and respect for unambiguous improvements, to infinite markets via the compactness theorem of propositional logic. In addition, we show that two versions of the lattice structure of finite markets carry over to infinite markets. At the same time, we prove that other results, such as weak Pareto optimality and strong stability property, do not hold in infinite markets. These results give us a clearer sense about which matching results are the most canonical.

Elliott Fairchild

Pointwise Temporal Decay of Solutions of the Klein-Gordon Equation in Schwarzschild Spacetime

under the direction of Ethan Sussman

Abstract

We investigate the time asymptotics of solutions of the free Klein-Gordon equation in Minkowski and Schwarzschild background spaces. We provide analytical methods to show that a $t^{-3/2}$ growth law holds in solutions of the equation over Minkowski spacetime and describe numerical methods that attempt to extend this law to radially symmetric solutions of the equation over Schwarzschild spacetime. We describe and explain examples where these methods fail to apply.

Gregory Li

Clique Structure of Orthomodular Posets

under the direction of Ethan Sussman

Abstract

Dacey graphs include looped structures of maximal cliques when two maximal cliques cover a third. Dacey established a class of simple, finite graphs, called Dacey graphs, which corresponds to the set of finite orthomodular posets, posets where the join of orthogonal elements exists and satisfy constructive properties. We begin an enumeration of Dacey graphs by presenting methods of partitioning Dacey graphs into equivalence classes based on the structures of maximal clique intersections and characterizing Dacey graphs which are edge-covered by m maximal cliques. This leads to a complete classification of Dacey graphs for $m \leq 4$. We then explore the implications of applying Dacey as a local condition to show each Dacey graph is locally-Dacey and investigate some bounds on such a local condition.

Andrei Mandelshtam

The Structure of $R_+(s)$

under the direction of Daniil Kalinov

Abstract

In this paper we study the semiring $R_+(s)$ for $s \in \mathbb{C}$. The inquiry into the structure of these objects is motivated by the study of functors between Deligne categories $\mathrm{Rep}(S_t)$ for $t \in \mathbb{C}\backslash\mathbb{Z}_+$. We prove that this semiring is a ring if and only if s is an algebraic number that is not a nonnegative integer. Furthermore, we show that all algebraic integers generated by s are also contained in this ring. We also make significant discoveries about the denominators of elements in $R_+(s)$ for s an algebraic integer, and characterize the primes dividing the denominators up to a finite set of primes. Moreover, we fully describe $R_+(s)$ for quadratic algebraic integers s.

Nikola Staykov

On the Sizes of Unions of Circles over Finite Fields

under the direction of Elia Portnoy

Abstract

This paper considers unions of circles over finite fields. We generalize an approach used by Oberlin, where in place of unions of lines we consider unions of circles. First we prove that there exists a dimension m, for which a generalized version of the so-called Wolff axiom holds for m-planes and m-spheres without any structural restrictions on the initial set of circles. Then we use this fact to make a construction of higher-dimensional spheres and planes in order to estimate the number of pairs of points on the circles from the initial set. We manage to obtain the bounds $P| \gtrsim |F|^{\frac{3}{2}d+\frac{\beta}{2}+\frac{1}{2}}$ and $|P| \gtrsim |F|^{\frac{3}{2}d+\frac{\beta}{2}+\frac{3}{4}}$ when the set of circles satisfies the d-Wolff axiom. Here $|P|$ is the number of pairs of points on the circles, $|F|$ is the size of the field and $d \in \mathbb{Z}$ and $-1 < \beta \leq 2$ are such that the number of circles in the initial set is $|F|^{3(d-1)+\beta}$.

Rahul Thomas

Width of Graph Mappings

under the direction of Elia Portnoy

Abstract

The *width* $|X/Y|$ of a topological space X over a topological space Y is the smallest integer k for which some continuous mapping $F : X \to Y$ has no point in Y with preimage greater than k. When X and Y are graphs, little is known about lower or upper bounds on the width. We investigate the width of complete graphs K_N over $m \times n$ grids $R_{m,n}$, as well as graphs with ε proportion of edges over general graphs G. First, we show the general lower bound $|X/Y| \geq (h(X)|V|)/(2|X/L|)$, where L is a line segment, $h(X)$ is the Cheeger constant of graph X, and $|V|$ is the number of vertices in X. Then, with a linear path projection, we show that $|K_N/R_{m,n}| = \Theta(N^2/\min(m,n))$, with the stricter bound $|K_N/R_{m,1}| \sim N^2/8$ for $m \times 1$ grids. Finally, we show that the width of a graph with N vertices and ε proportion of edges over a graph with V vertices is $\Omega(\varepsilon^2 N^2)$ for $\varepsilon \leq 2/V$ and $\Omega(\varepsilon^2 N^2/V)$ otherwise.

Gerald Xu

Geometric Quaternionic Quantum Mechanics

under the direction of Ethan Sussman

Abstract

In quantum mechanics, the symmetries of a physical system are closely related to the conservation laws within that system. As a result, a mathematical understanding of a system's symmetries allows us to accurately model and describe that system, and therefore we want to find methods of mathematically representing these symmetries. Past works by Eugene Wigner and Daniel Freed have proved methods of representing symmetries as complex matrices and complex projective spaces, respectively. This project closely follows the work of Daniel Freed, but with respect to quaternionic projective spaces. It is known that symmetries in physical systems preserve empirical quantities. In both Freed's and our project, we are interested in the symmetries that preserve transition probabilities, which are the probabilities that a state of the system would evolve to other states.

Our work focuses on quaternionic projective spaces, and proving the quaternionic analog to Freed's work. We give the quaternionic analog to the Fubini-Study metric on complex projective space, as well as describe its associated distance function. We prove the relation between this distance function and transition probability, demonstrating that symmetries in quantum mechanics can be represented by isometries in quaternionic projective space.

Alexis Yi

Bounding The Size of The Modular Pants Graph

under the direction of Catherine Wolfram

Abstract

The pants graph was first introduced to better understand distinct pants decompositions of hyperbolic surfaces. The action of the mapping class group on the pants graph gives rise to a modular pants graph, which allows for the study of homeomorphism classes of pants decompositions. Using the bijection between homeomorphism classes of pants decompositions and trivalent multigraphs, we provide two upper bounds on the number of vertices in the modular pants graph of genus g: the first is in terms of the number of trivalent multigraphs of genus g with one bridge; the second is a recursive bound in terms of g which makes use of girth.