Dimitar Chakarov

On a Generalization of Artin's Conjecture on Primitive Roots

under the direction of Yichi Zhang

Abstract

We propose a generalization of Artin's conjecture on primitive roots to the ring $\mathbb{Z}[i]$ of Gaussian integers. We conjecture that for a fixed $q \in \mathbb{Z}^+$, every $a \in \mathbb{Z}[i] \setminus \{\pm i, 0, \pm 1\}$ generates a cyclic subgroup of the multiplicative group $(\mathbb{Z}[i]/\mathfrak{p})^\times$ of index $[(\mathbb{Z}[i]/\mathfrak{p})^\times : \langle a \rangle/\mathfrak{p}] = q$ for infinitely many prime ideals $\mathfrak{p}$. We prove the conjecture when $a \in \mathbb{Z}$, and in several special cases reduce it either to the classical Artin's conjecture, or to its extension for near-primitive roots, the Golomb's conjecture. We conclude by showing that for every integer a, we have $\sum_{q=1}^{\infty} \delta_{a,q} = 1$, where $\delta_{a,q}$ is density of the prime ideals $\mathfrak{p}$ yielding subgroups of index precisely q.

Lauren Chen

Two Problems on Cantor Set Arithmetic

under the direction of Yuqiu Fu

Abstract

We find the cardinalities of the solution sets to the polynomial equations $c = a + b$ and $c = a - b$ on variants of the Cantor set. We also compute examples for the equation $c = ab$. A previous theorem states $f(C \times C) = [0, 1]$ for the Cantor set C where $f(x, y) = x^2y$. Our second problem generalizes this to $f = x^\alpha y$ for α in the range $\frac{\log 2}{\log 3/2} \leq \alpha \leq 2$. We also explore the case when α is greater than 2. We consider the expansion of $f(C_n \times C_n)$ for a few small n, where C_n is the nth iteration of the Cantor set, to find intervals of $\alpha > 2$ such that $f(C \times C)$ does not cover the entire interval $[0, 1]$.

Eddie Hu

$R_+(S)$ for Algebraic Numbers

under the direction of Daniil Kalinov

Abstract

In this paper, we try to calculate $R_+(S)$ for algebraic numbers S in order to show the existence of symmetric monoidal functors between Deligne categories $\text{Rep}(S_t) \to \text{Rep}(S_t')$. We first evaluate $R_+(S)$ for rational S ($R_+(S) = \mathbb{Z}[\frac{1}{n}] = \mathbb{Z}[\frac{m}{n}]$). In the case where $S = \sqrt{n}$, we use the gained intuition from the rational case as well as quadratic reciprocity to reach certain conclusions about the possibilities of $R_+(S)$. We not only get $\mathbb{Z}[\sqrt{n}] \subset R_+(\sqrt{n})$, but also the stronger statement for $n = 2$ that $R_+(\sqrt{2}) = \mathbb{Z}[\frac{1}{p_1}, \frac{1}{p_2}, \ldots, \frac{1}{p_i}, \ldots][\sqrt{2}]$ for primes p_i such that $\left(\frac{2}{p_i}\right) \neq 1$.

Benjamin Kang

A Variation of the Erdös-Turán Conjecture and the Minimality of Multiplicative Bases

under the direction of Yichi Zhang

Abstract

One problem that has been unsolved for nearly a century is the Erdös-Turán Conjecture, an important problem in additive number theory. It states that the efficiency of additive bases of order two of positive integers is always infinite. In this paper, we work towards a solution to the multiplicative analog of this problem. First, we prove that the Erdös-Turán Conjecture implies its multiplicative analog. Then, we introduce the density of a set and prove that finite efficiency is only possible if the basis has density zero. Furthermore, we provide examples of bases and calculate their densities, where one of our bases has density zero. Lastly, we consider a partially ordered set of the bases, and we consider which bases are minimal elements in this set. If a basis is not minimal, we attempt to discover the smallest subset which remains a basis along with its density.

Rupert Li

$\mathbb{F}_q$-rational points of hypersurfaces in weighted projective spaces over finite fields

under the direction of Chun Hong Lo

Abstract

We investigate $\mathbb{F}_q$-rational points on hypersurfaces in weighted projective spaces over the finite field $\mathbb{F}_q$. Particularly, we consider the maximum number of $\mathbb{F}_q$-rational points that can lie on a hypersurface of a given degree, weighted projective space, and finite field. In classical projective space, Serre answered this question by proving Serre's inequality. We provide conjectures generalizing Serre's inequality to weighted projective spaces and prove some partial results. We also prove which values the number of $\mathbb{F}_q$-rational points on a given hypersurface can take, and give some further conjectures about these possible values and their distributions.

Jason Liu

On Q-binomial Polynomials and Quantum Integer-Valued Polynomials

under the direction of Robert Burklund

Abstract

We study rings of compactified q-deformed integer-valued polynomials defined by Harman and Hopkins, along with another ring of multi-variable q-deformed integer-valued polynomials. We disprove a conjecture of theirs about how one such compactified ring may be generated andfind a basis for a generalization of these sets of polynomials. We also find a basis for the multi-variable analog of these polynomials.

Austen Mazenko

Determination of Markov Model Dynamics from Equilibrium Data Snapshots

under the direction of Dominic Skinner

Abstract

Often, when using a Markov State Model (MSM) to model a physical or biological system, only the equilibrium distribution is experimentally measurable, yet this equilibrium alone is insufficient to uniquely fix the system's transition probabilities. To determine these probabilities and thus the dynamics of such systems, this paper considers inhibiting various transitions and using the new equilibria to gain information about the system. We completely determine the minimum number of cuts required to fully characterize three-state systems, and conjecture that $n-1$ cuts is both necessary and sufficient for complete, n-state systems. Because such a characterization is inherently valid only up to scaling, we establish the number of blocks in the transition graph as a lower bound on the degrees of freedom. Finally, we simulate systems to confirm the practicality of our minimum-cut algorithm for the three- and four-state situations.

Assylbek Olzhabayev

Block diagonal form of the Varchenko matrix of Oriented Matroids

under the direction of Adela YiYu Zhang

Abstract

The definition of the Varchenko matrix for hyperplane arrangements, introduced by Alexandre Varchenko in 1993, extends naturally to oriented matroids. We generalize the theorem of Gao and Zhang by proving that the Varchenko matrix of an oriented matroid has a diagonal form if and only if the pseudohyperplane arrangement corresponding to the oriented matroid is in semigeneral position, i.e. it does not contain a degeneracy. We also show that the Varchenko matrix of a pseudoline arrangement with one degeneracy has a block diagonal form and apply it for proving the Varchenko matrix determinant formula.

AnaMaria Perez

On Generalizations of the Double Cap Conjecture

under the direction of Rose Zhang

Abstract

The problem of finding the largest $\frac{\pi}{2}$-avoiding spherical set first appeared in the AMS monthly journal in 1974. H. S. Witsenhausen asked readers to determine $\alpha(n) = \dfrac{|U|}{|S^{n-1}|}$, where U is the largest $\pi/2$-avoiding set in the n dimensional sphere S^{n-1}, that is, the largest set on the sphere in n dimensions containing no orthogonal vectors. We look at a variation of this problem: determine $\alpha(n,k) = \dfrac{|U|}{|S^{n-1}|}$, where U is the largest set on the n dimensional sphere S^{n-1} that contains no k mutually orthogonal vectors. We prove a lower bound for $\alpha(n,k)$ for any $n \geq 3$. We specialize to the case $n = k = 3$ by considering spherical sets that avoid three mutually orthogonal vectors in 3D in an attempt to determine $\alpha(3,3)$ which allows for more interesting configurations and visualization. We find the largest spherical subset U not containing three mutually orthogonal vectors in the following cases: (i) U is a large double cap; (ii) U is a centered band; or (iii) U is a wedge shape. We also find a good lower bound for the maximal area in the case that U is the union of two double caps and a centered band. The measures of these sets give a lower bound for $\alpha(3,3)$. Additional computational results suggest a configuration that gives an even better lower bound for $\alpha(3,3)$ as well as a possible relationship to the moving sofa problem.

Alexander Zhu

Modeling Local Order in Bacterial Collective Motions

under the direction of Boya Song

Abstract

In our study, we modify an existing dynamic model of how the position and orientation of cells change as a result of their self-propulsion speed and cell-cell repulsion forces. We add a force between endpoints to create smectic alignment in the simulations in addition to nematic alignment. We use a MATLAB code to run simulations of different scenarios of cell interactions to test our improved model. Our modified model succeeds in creating smectic alignment in the two cell scenario when the cells are aligned in the same direction and when two cells are oppositely oriented with low velocities. In the 10 cell scenario, our model succeeds in creating smectic alignment when the cells have the same orientation and velocity, and they create rafts among other cells with similar speeds and orientation. We simulate scenarios with 50 cells with random orientations and positions, and our model succeeds in creating curved and linear endpoint alignment in these scenarios. Our model generates endpoint alignment when the cells have similar orientations and velocities, creating rafts similar to those seen in previous experiments.