

Research Science Institute (RSI)
Mathematics Section
Abstracts of Final Research Papers
M.I.T.
Summer 2015

Amy Chou

NP-Complete Triangle Packing Problems

under the direction of Mr. James Hirst

Abstract

Triangle packing problems ask whether a collection of given triangular pieces can be placed inside a target region without overlap. We show that general triangle packing – packing arbitrary triangular pieces inside a target region – is NP-complete. Additionally, we show that three specific instances of the general triangle packing problem are NP-complete: packing right triangles into a rectangle, packing right triangles into a right triangle, and packing equilateral triangles with fixed orientation into disconnected parallelograms. We prove the hardness of these problems by reducing from the known NP-complete problems 3-partition and 4-partition.

Brandon Rafal Epstein

Maximizing the Number of Lattice Points on a Strictly
Convex Curve

under the direction of Hong Wang

Abstract

We obtain an upper bound for the maximum number of lattice points that exist on the graph of a smooth convex function $f : [0, N] \rightarrow [0, N^\gamma], \gamma > 0$. This bound could serve as an estimate of the area bounded by a convex curve.

Kristian Georgiev

On the size of unions of lines in \mathbb{F}^n satisfying the Wolff
axiom

under the direction of Hong Wang

Abstract

The paper deals with finding unions of lines in $\mathbb{F}_{p^{2k}}^n$ which obey the Wolff axiom and have minimal size. We provide an extension of the constructions for $\mathbb{F}_{p^{2k}}^3$, obtained by Tao in 2002, to a construction for $\mathbb{F}_{p^{2k}}^n$. We determine the size of the union of lines in our construction in $\mathbb{F}_{p^{2k}}^n$ to be $O(p^{1.6kn})$. We prove that our construction obeys the Wolff axiom up to a heuristically negligible number of lines. The next step would be to prove rigorously that this number is indeed negligible.

Brian Gu

Bounds on Maximal Size Tournament Codes

under the direction of Mr. Brandon Tran

Abstract

We compute new bounds on the maximal size of tournament codes. We improve the current upper bound by a constant factor in the exponent, and demonstrate a conditional method for improving the upper bound based on the number of 0s and 1s in the optimal tournament code. Lastly, we consider the related problem of covering a complete graph with bicliques of bounded size. We find and prove an asymptotic formula for the minimum number of bicliques needed, up to a small constant factor.

Dylan Hendrickson

Generalizing the Inversion Enumerator to G-Parking
Functions

under the direction of Pavel Galashin

Abstract

We consider a generalization of the inversion enumerator for G -parking functions, $I_G(q)$. We find several recurrences for $I_G(-1)$, including a recursive formula whenever G is the cone over a tree. We relate $I_G(-1)$ to the number of partial orientations of G . Using the connection between I_G and the Tutte Polynomial, we find another recursive formula for $I_G(-1)$. For any partial cone G over a tree T , we compute $I_G(-1)$ by counting partial orientations of T with a specific set of vertices with even in-degree.

Meena Jagadeesan

An Analytic Approach to the Modeling of Cell Motion
in Porous Media

under the direction of Mr. Andrew Rzeznik

Abstract

Swimming cells play important roles in human health through reproduction and the spread of infection from biomedical devices and in ecology through the recycling of organic matter. Much research has been done to study cell motility in bulk fluids. However, the environments of many swimming cells are ripe with porous, microstructured materials and boundaries, which significantly affect cell motion. The goal of the project is to establish a quantitative framework that accurately models key aspects of microbial locomotion in porous media. In this study, we focus on an analytic approach that complements the lab experiment and computer modeling components of the other project members.

Cell motion is conventionally modeled microscopically as a simple random walk or as run-and-tumble motion. We consider the intermediate model of a random walk with one-step memory on the dual graph defined by the fluid pockets of a porous medium. In the case of low particle density, we prove that for a large class of n -dimensional porous media, the stationary distribution can be broken into $n + 1$ classes. Within each class, the stationary distribution is local, while between classes, there can be global shifts in probability density. Furthermore, we consider the effect of merging porous media through the addition of tunnels. We also present our initial work on generalizing the low density case to higher density, where cells are strongly affected by particle-particle interactions.

Emily Jia

Extremal Number of Trees in Hypercubes

under the direction of Mr. Chiheon Kim

Abstract

Given a tree T , we investigate bounds on the extremal number of T in the hypercube Q_d , defined as the maximum number of edges in a subgraph X of Q_d that is T -free. We first present an analog of the Erdős-Sós conjecture in the hypercube and compare the extremal numbers of stars and similar structures to this bound. We provide examples of families of trees that achieve this lower bound and others whose extremal number is almost twice this bound. From there, we demonstrate a restriction on minimum degree that guarantees the existence of certain trees in subgraphs of the hypercube.

Abijith Krishnan

A Secretary Problem with a Sliding Window for
Recalling Applicants

under the direction of Dr. Shan-Yuan Ho

Abstract

Data stream mining is a tool to increase the amount of data that can be collected and to identify the most important data point in a stream of real-time data. The Classical Secretary Problem addresses the optimal strategy to choose the maximum element of a sequence with the constraint that once an element is skipped, it becomes unavailable. To make the Classical Secretary Problem more applicable to data stream mining, we offer a Sliding Window of choices and allow any of the last K data points to be chosen. We characterize the probability of winning in two cases of the Sliding-Window Secretary Problem analytically and examine the limiting cases of this problem.

Wilbur Li

Distributions of the k -major index in random words and
permutations

under the direction of Cesar Cuenca

Abstract

The most noted permutation statistics, inversion number and major index, have been studied extensively in recent decades. Many results found regarding their joint distribution and normality have been found. We focus on the k -major index, a natural interpolation between the inversion number and major index, which exhibits elegant properties of equidistribution. We generalize normality and equidistribution to words with fixed alphabet size and large length. We then draw analogs between the fluctuation of the k -major index on a random, fixed permutation or word as k varies and standard Brownian motion. Finally, we observe limiting shapes of joint distributions as permutation length increases without bound and propose a generating function technique to characterize k -majors for small k .

Sarvasva Raghuvanshi

Ramsey Multiplicity and Commonality of Graphs

under the direction of James Hirst

Abstract

Let us define a *copy* of a graph G inside a larger graph K to be a subgraph $H \subset K$ on $|V(G)|$ vertices such that $G \subseteq H$. Similarly, we define an *anticopy* of G to be a subgraph $H \subset K$ on $|V(G)|$ vertices such that $G \subseteq \bar{H}$. A graph F is common if and only if the minimum density of copies and anticopies of F in any graph G is $2^{1-|E(F)|}$. Note that this minimum is attained when G is a random graph with edge density one-half. We provide a proof, using flag algebras, that the graph obtained from a pentagon by adding a chord is common. We also propose a modern proof that the graph formed from any number of disjoint copies of a common graph is itself common and open some other questions about the commonality of disjoint graphs.

Pol Gómez Riquelme

A combinatorial interpretation of the h - and γ -vectors
of the cyclohedron

under the direction of Pavel Galashin

Abstract

We give an overview of the theory of convex polytopes and focus our attention on the family of graph-associahedra, constructed from connected graphs. We present a combinatorial interpretation of the γ -vector of the cyclohedron and conjecture a descent-preserving bijection between the \mathcal{B} -trees of the cycle graph on $n + 1$ vertices and all lattice paths on the $n \times n$ grid.

Uma Roy

An Explicit Description of the $\mathcal{B}(\infty)$ Crystal for
Generalized Quantum Groups of Comet Quivers

under the direction of Seth Shelley-Abrahamson

Abstract

To a quiver one can associate an algebraic object known as a quantum group and much effort has been dedicated to studying the negative part of the quantum group with the theory of crystal bases. Analysis of the combinatorial structure of crystal bases has provided significant insight into the algebraic structure of quantum groups in classical types. Recently, Tristan Bozec gave a definition of *generalized quantum groups* that extends the usual definition of quantum groups to quivers with loops at vertices. We explicitly characterize the crystal $\mathcal{B}(\infty)$ associated to a generalized quantum group corresponding to a certain type of non-isotropic comet quiver. Our result provides a description of when two sequences of Kashiwara operators are equal by establishing a family of relations amongst the Kashiwara operators themselves.

Jared Tramontano

The Status Update Problem: Optimal Strategies for
Pseudo-Deterministic Systems

under the direction of Dr. Shan-Yuan Ho and Mr. Siddharth
Venkatesh

Abstract

We investigate First-Come-First-Served (FCFS) systems receiving update packets from a general integer number of sources. We calculate precisely when these sources should be sending update packets, relative to the rate at which the system can deterministically process update packets, to keep the average age of information, Δ , minimal. We devise strategies specifically for a single source, so that the server is never idle, so that the server has some idle time after every update packet, and so that the strategy is optimal in general. Finally, we discuss the case in which the server processes packets as a random process.