Limit Laws for Coefficients of Polynomial Powers

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MIT-PRIMES

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A warm-up

$$(1+x)^n = 1 + nx + \binom{n}{2}x^2 + \cdots + x^n.$$

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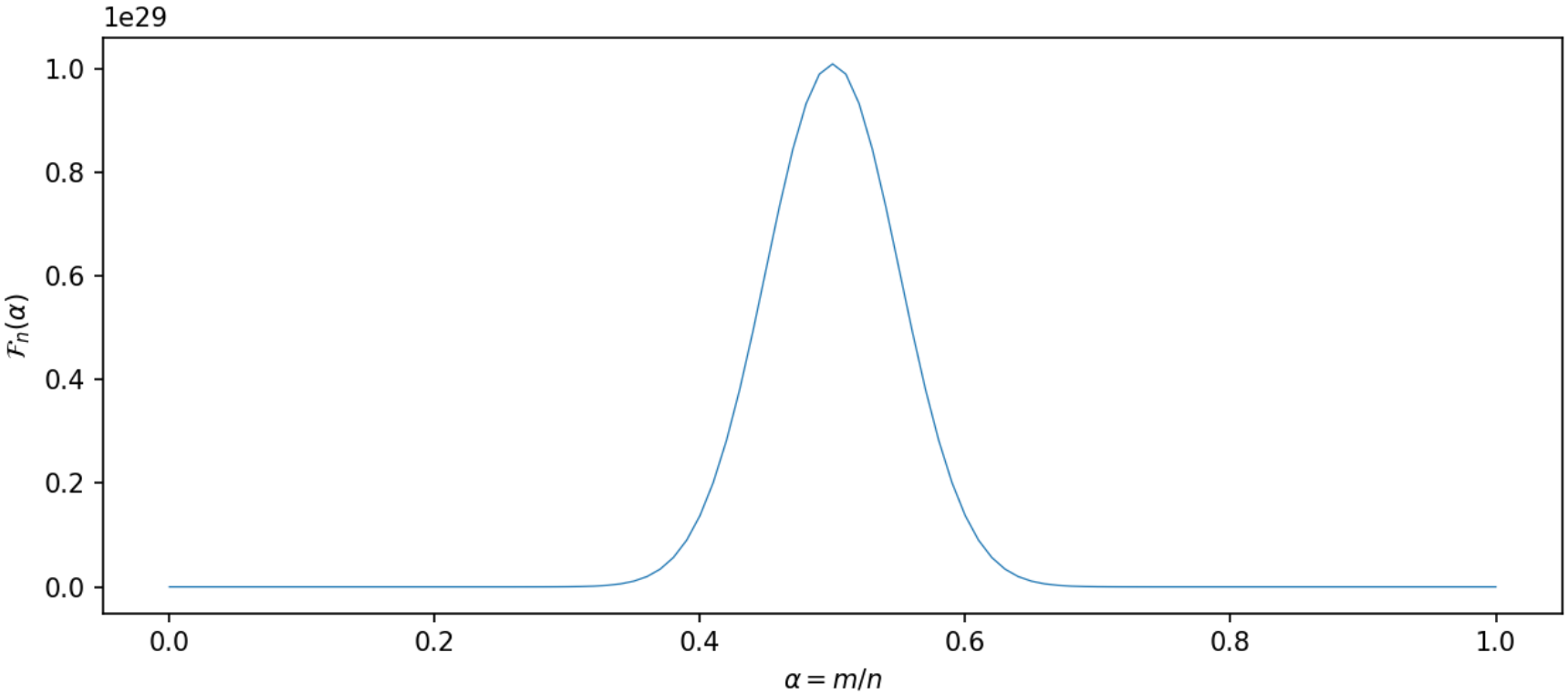


Figure: Coefficients $[x^m](1+x)^n$, with $n = 100$.

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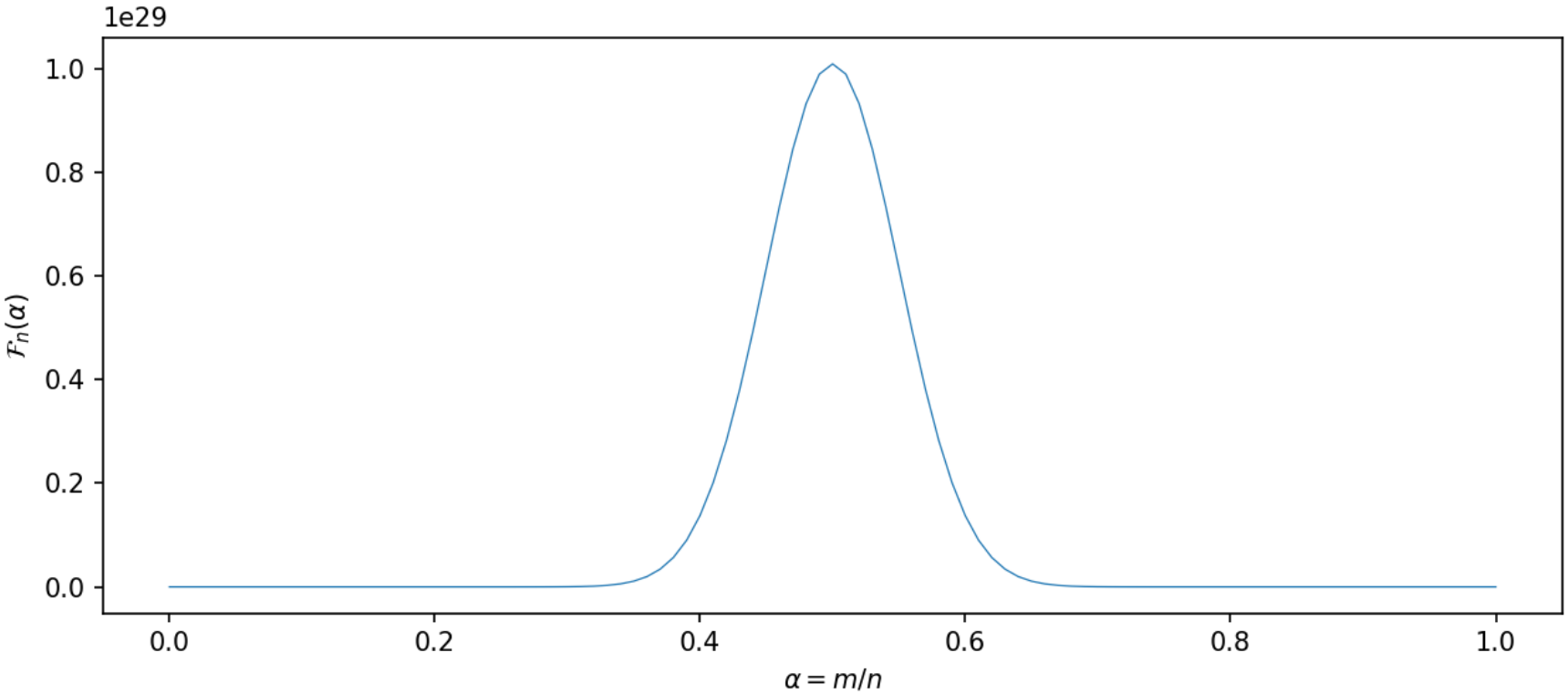


Figure: Coefficients $[x^m](1+x)^n$, with $n = 100$.

As $n \to \infty$, the graph is well-approximated by a (scaled) Gaussian centered at $\alpha = 1/2$ with variance $\frac{1}{4n}$ in α.

Another simple example

Pick $P(x) = 1 + x + 3x^2$.

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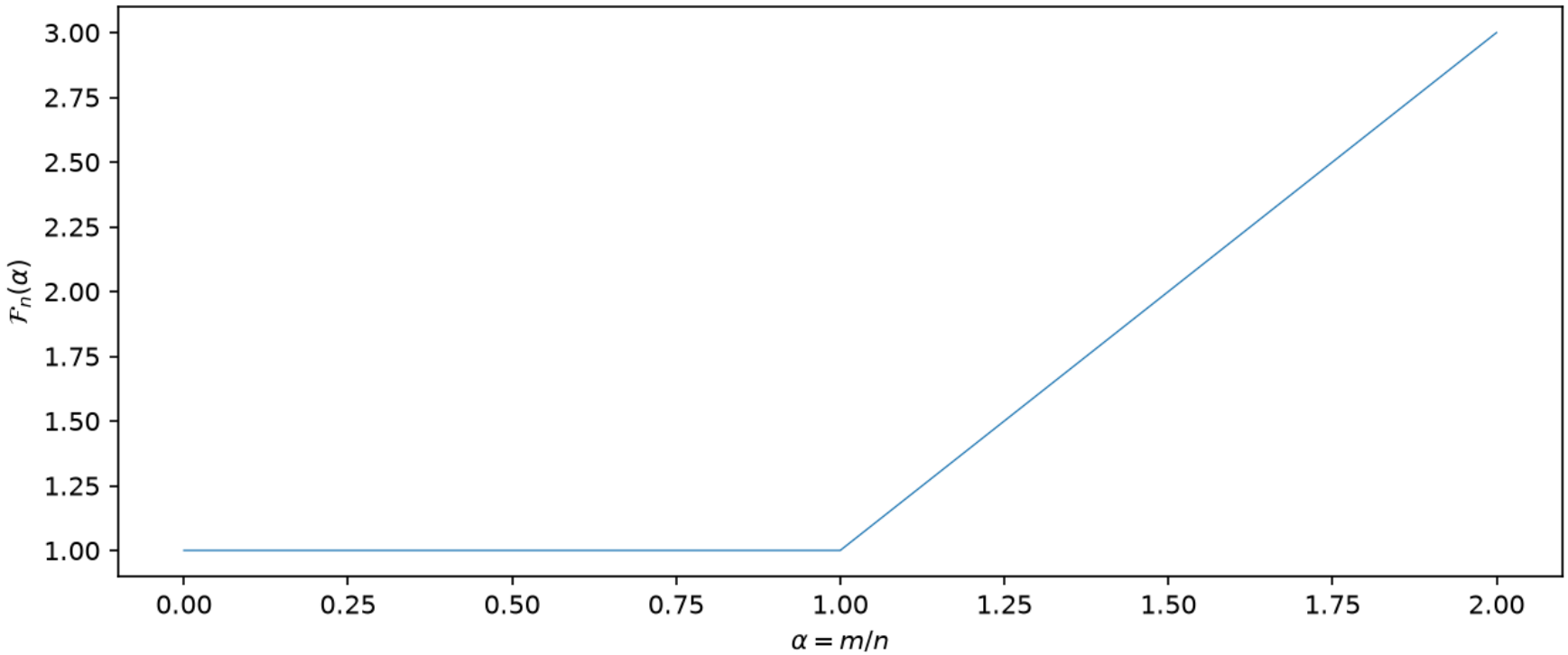


Figure: Coefficients $[x^m]P(x)^n$; $P(x) = 1 + x + 3x^2$, $n = 1$.

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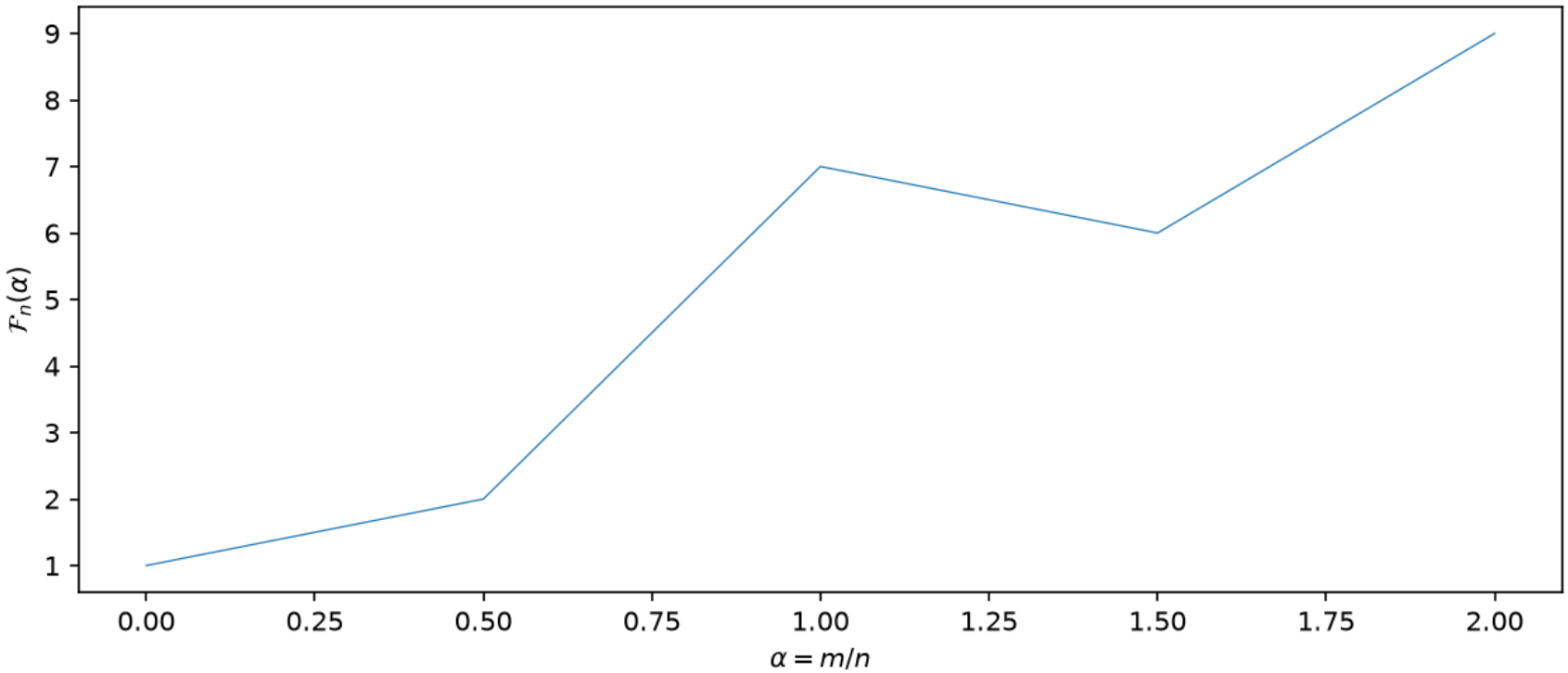


Figure: Coefficients $[x^m]P(x)^n$; $P(x) = 1 + x + 3x^2$, $n = 2$.

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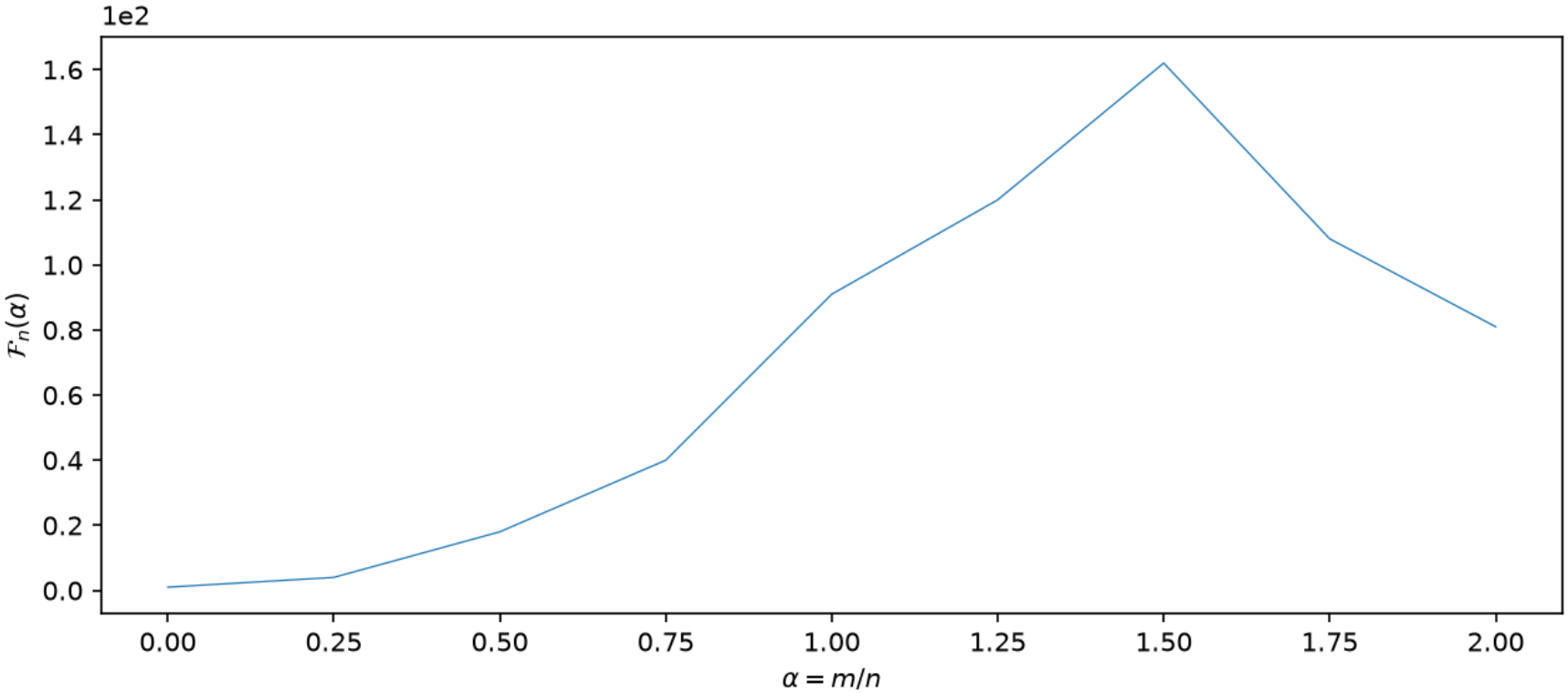


Figure: Coefficients $[x^m]P(x)^n$; $P(x) = 1 + x + 3x^2$, $n = 4$.

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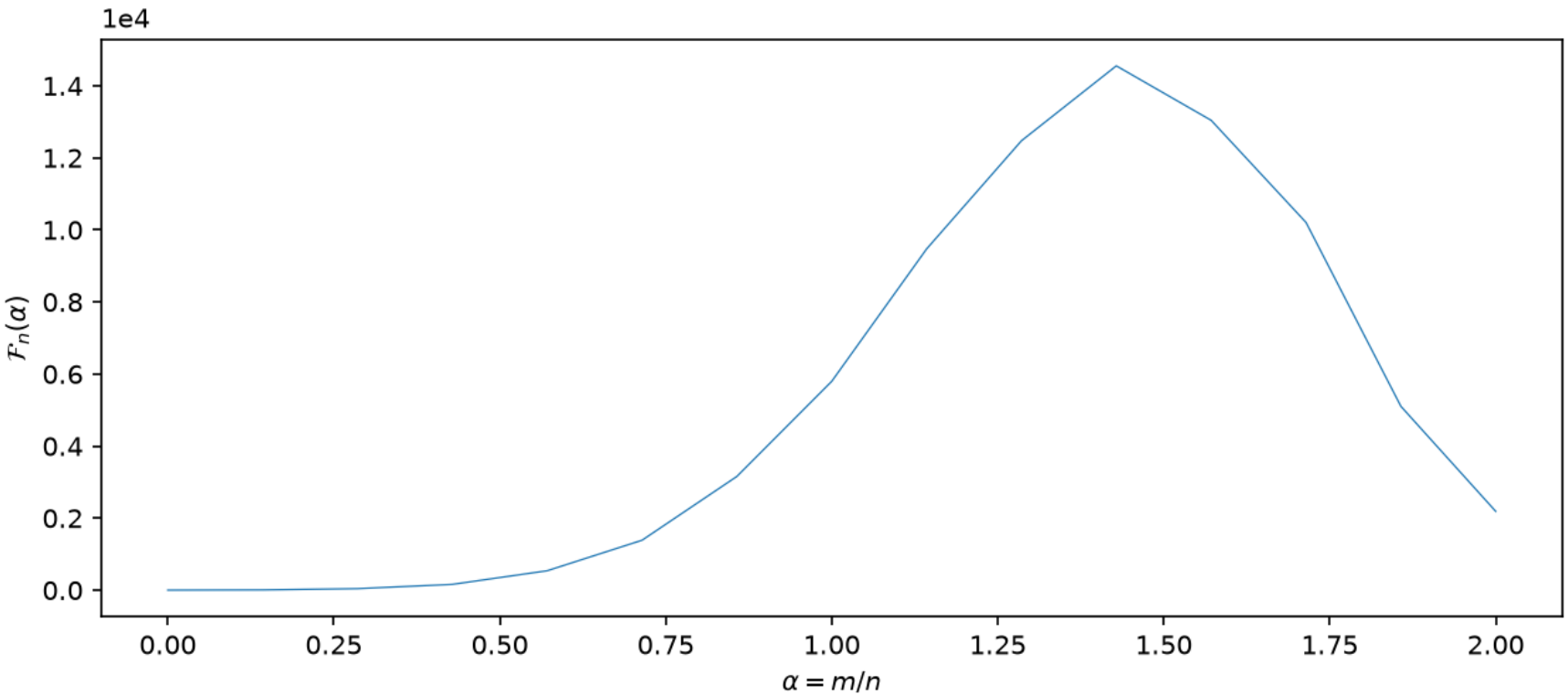


Figure: Coefficients $[x^m]P(x)^n$; $P(x) = 1 + x + 3x^2$, $n = 7$.

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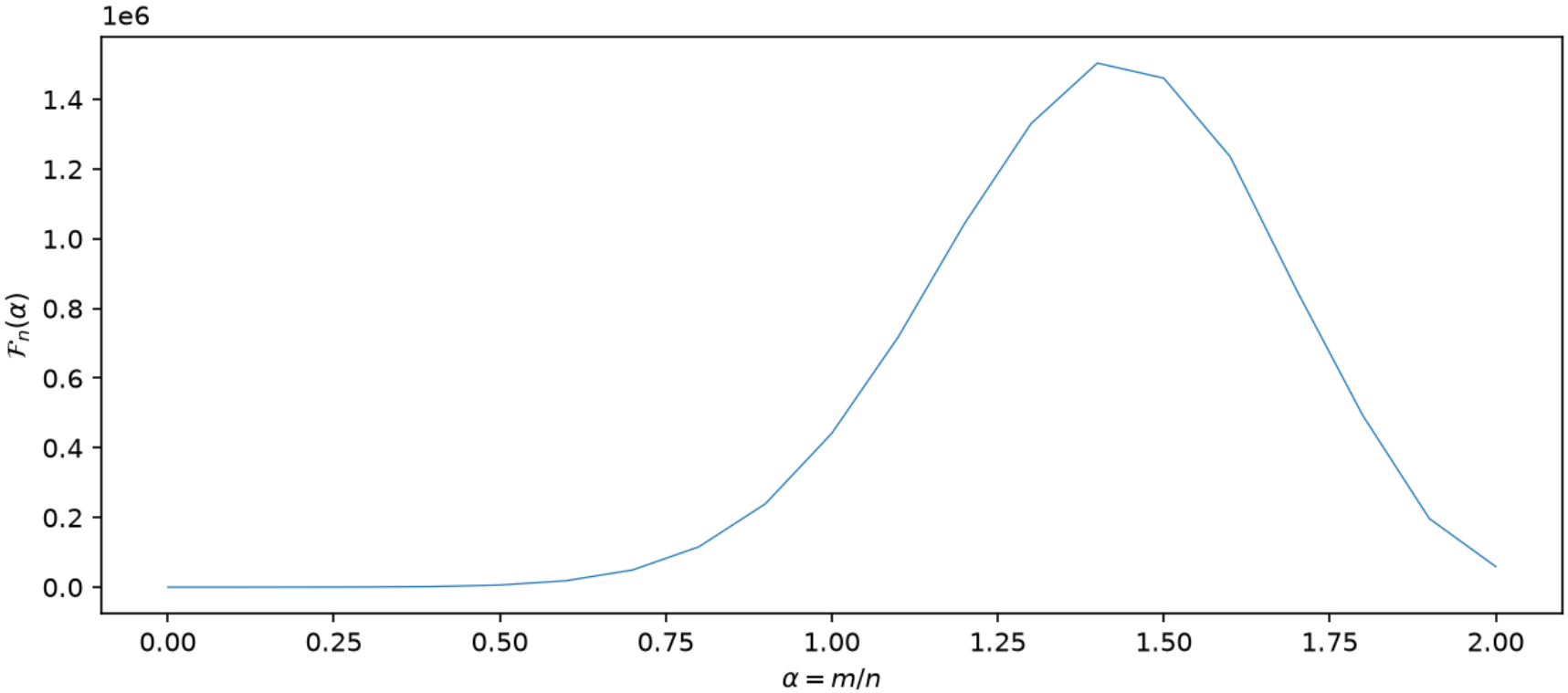


Figure: Coefficients $[x^m]P(x)^n$; $P(x) = 1 + x + 3x^2$, $n = 10$.

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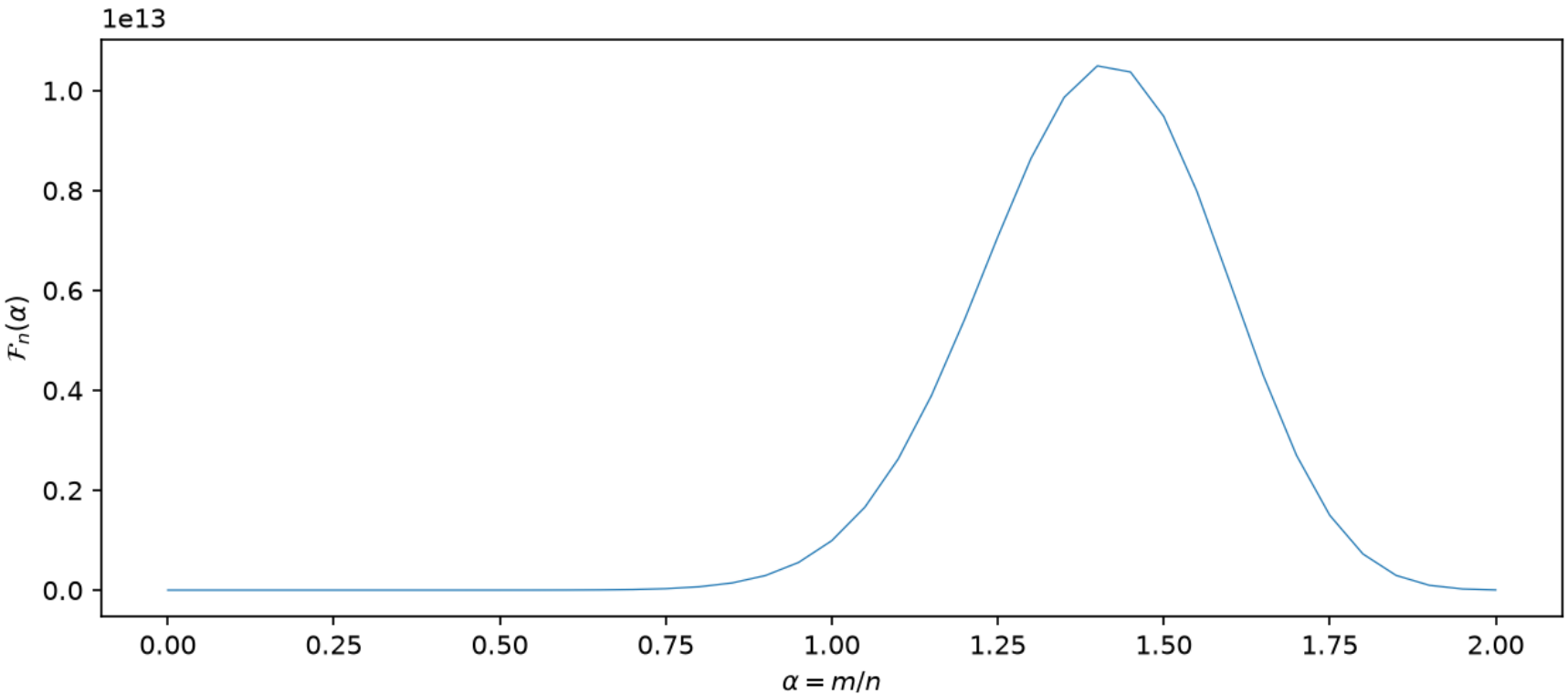


Figure: Coefficients $[x^m]P(x)^n$; $P(x) = 1 + x + 3x^2$, $n = 20$.

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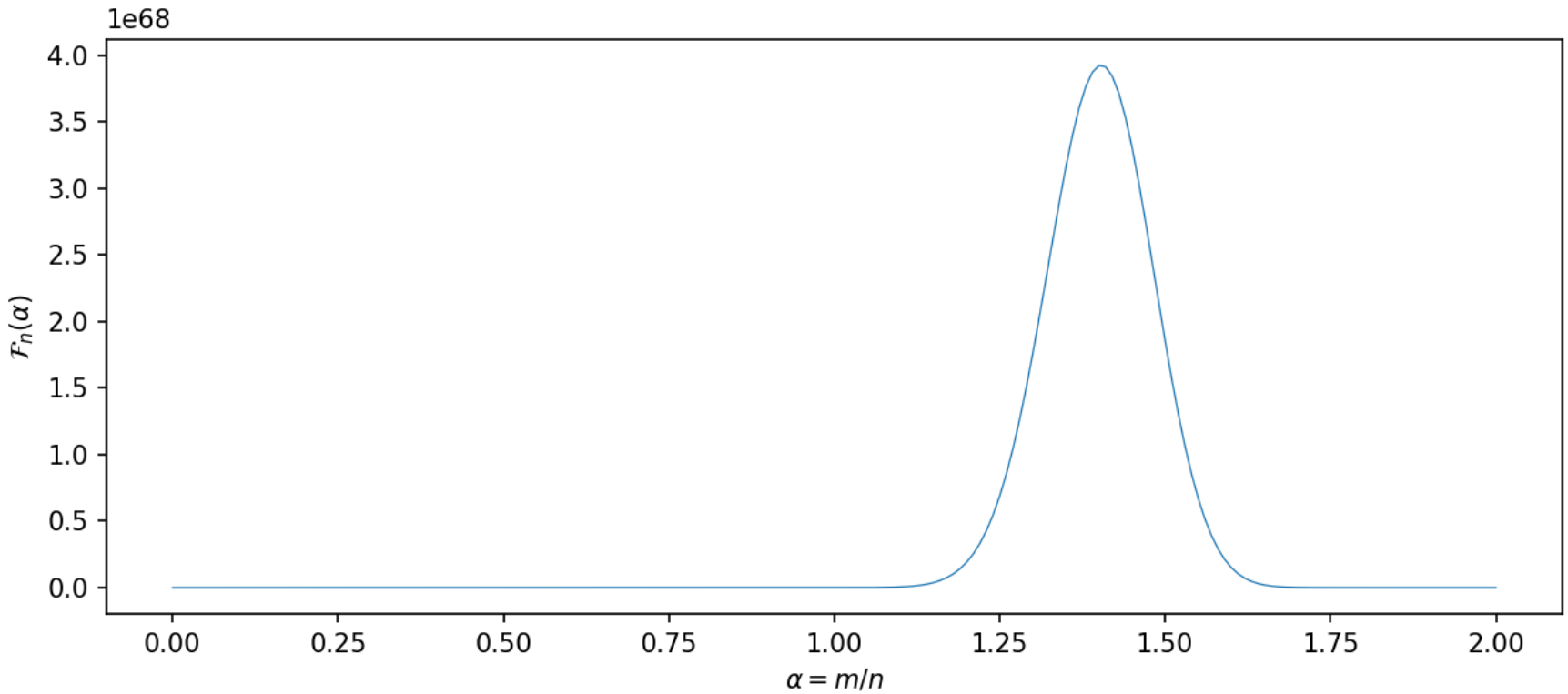


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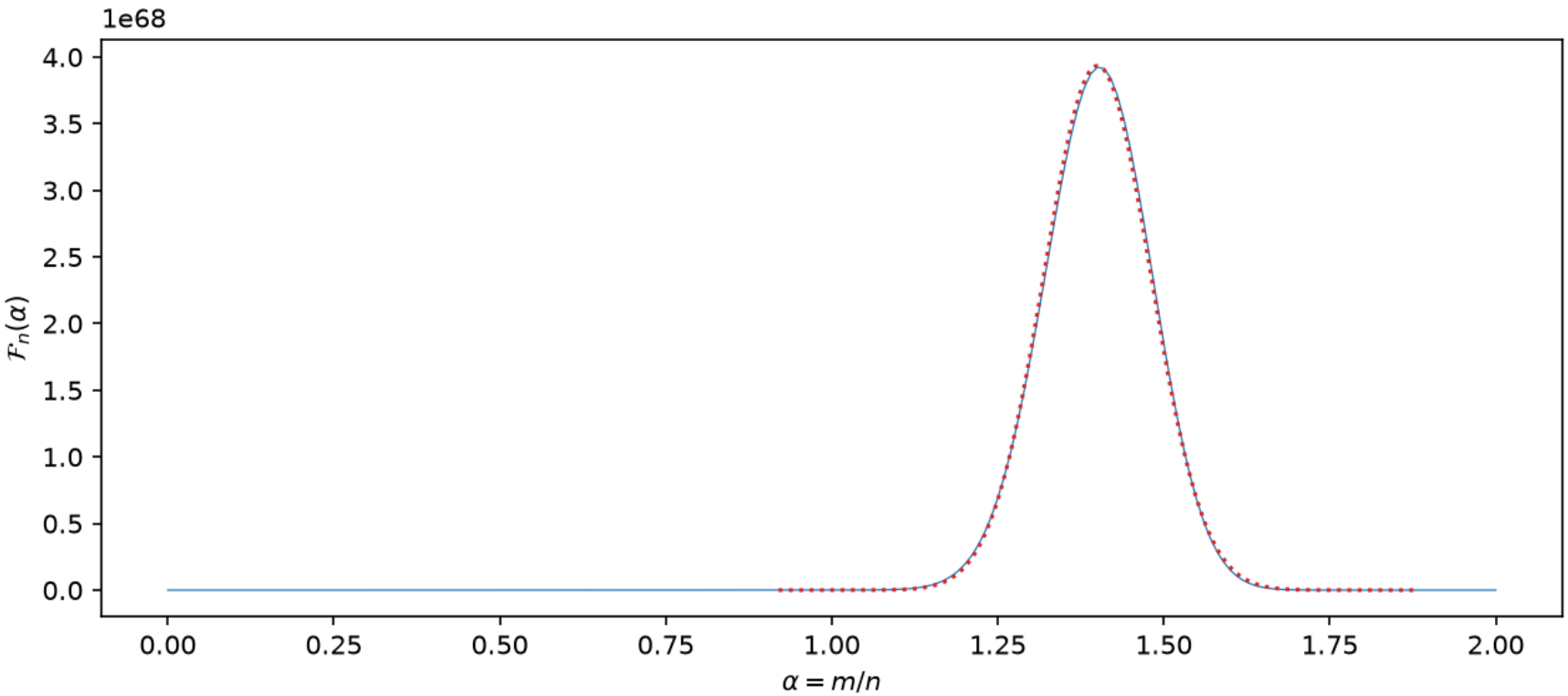


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As $n \to \infty$, the graph is well-approximated by a (scaled) Gaussian centered at $\alpha = 7/5$ with variance $\frac{16}{25n}$ in α.

Another motivating example

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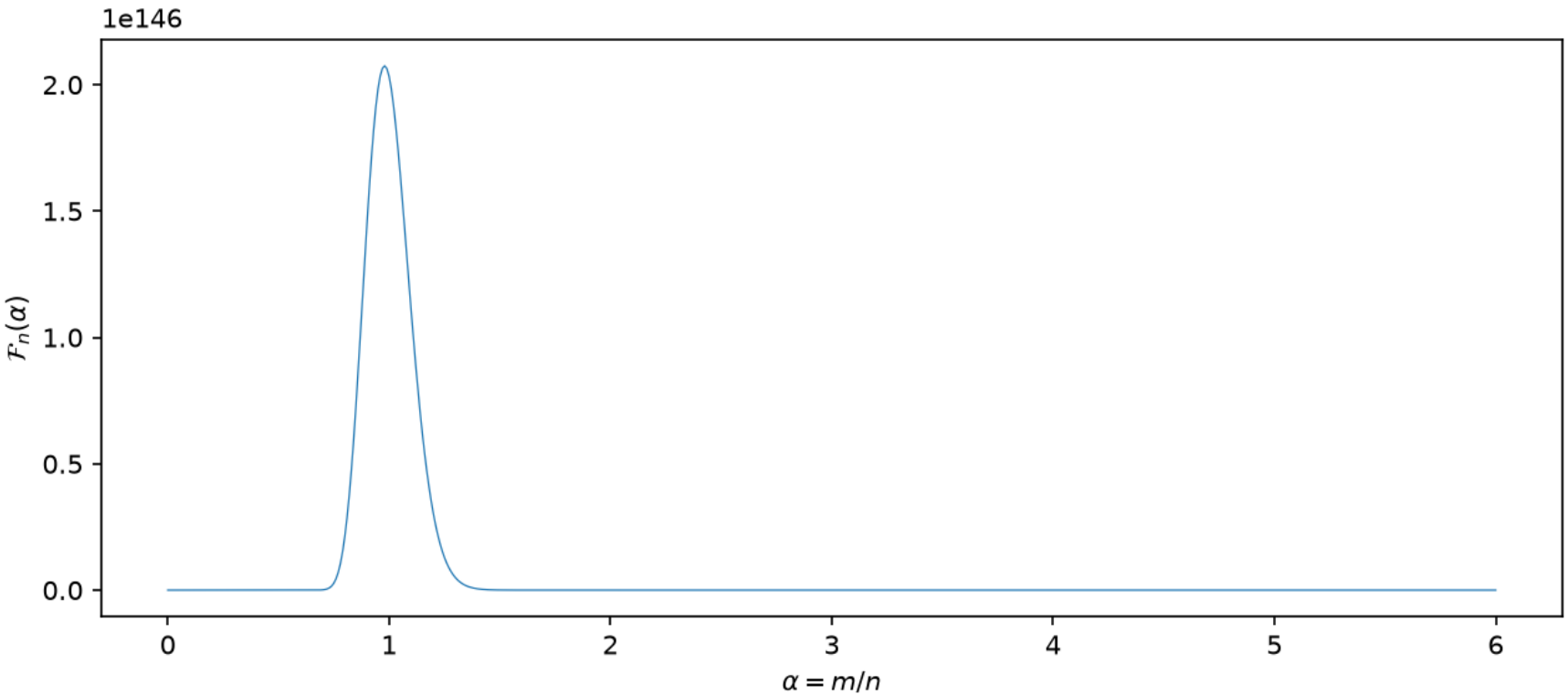


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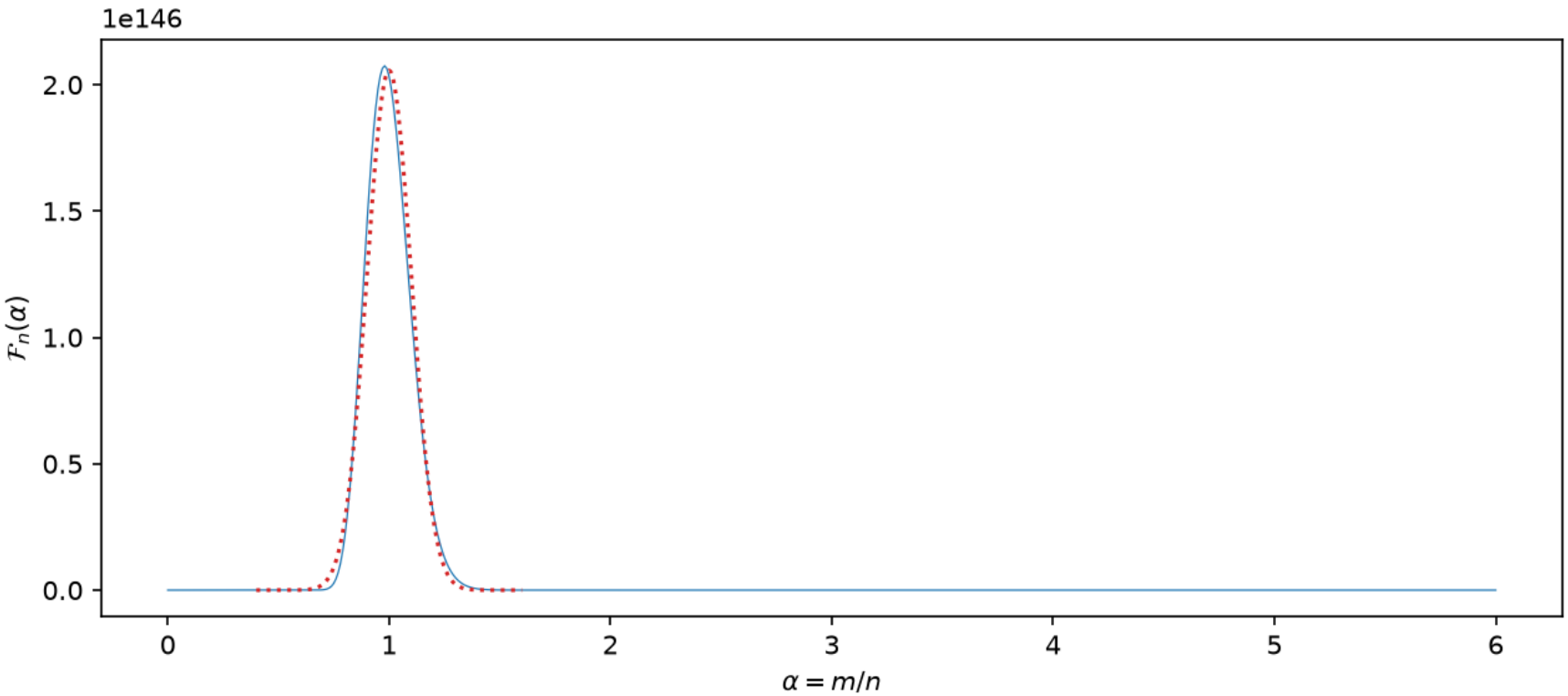


Figure: Coefficients $[x^m]P(x)^n$, with $n = 100$.

As $n \to \infty$, the graph is well-approximated by a (scaled) Gaussian centered at $\alpha = 1$ with variance $\frac{1}{n}$ in α.

Multiple peaks

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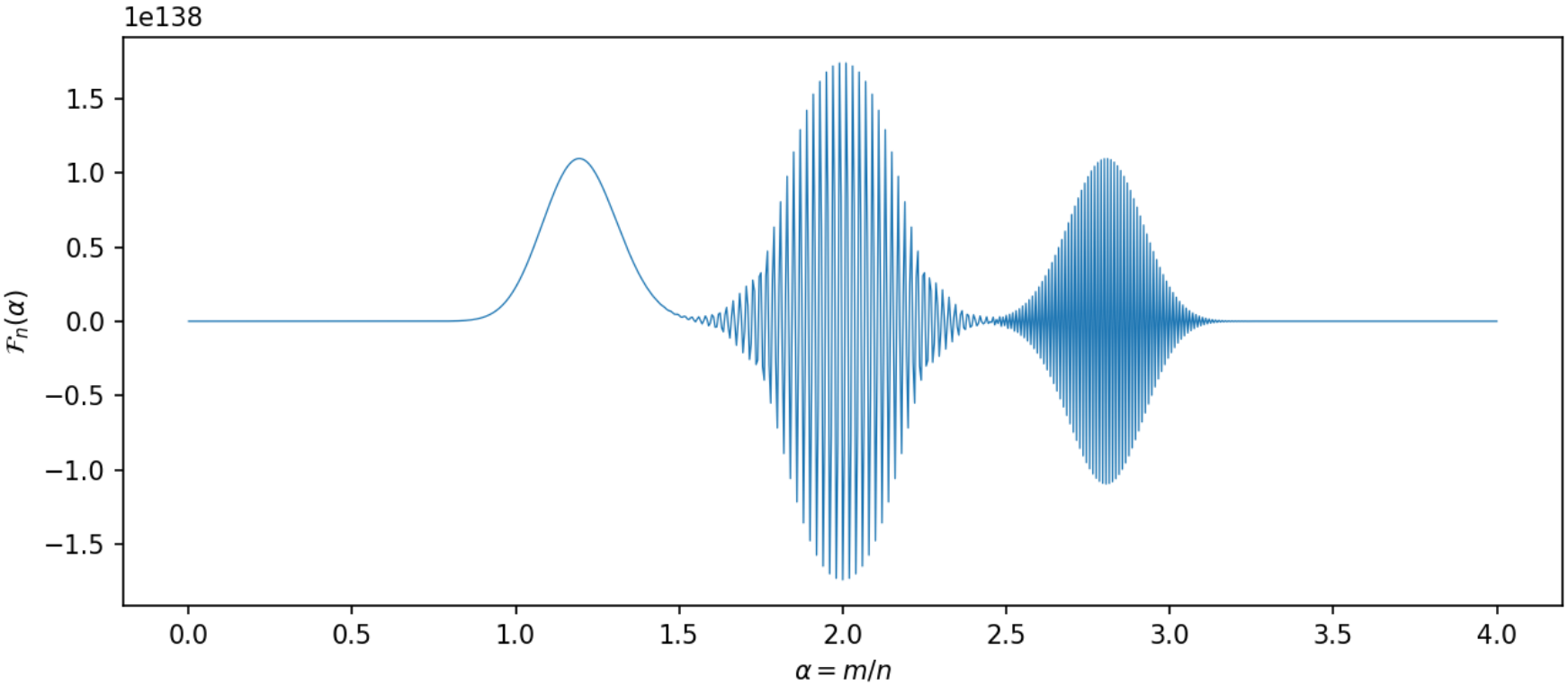


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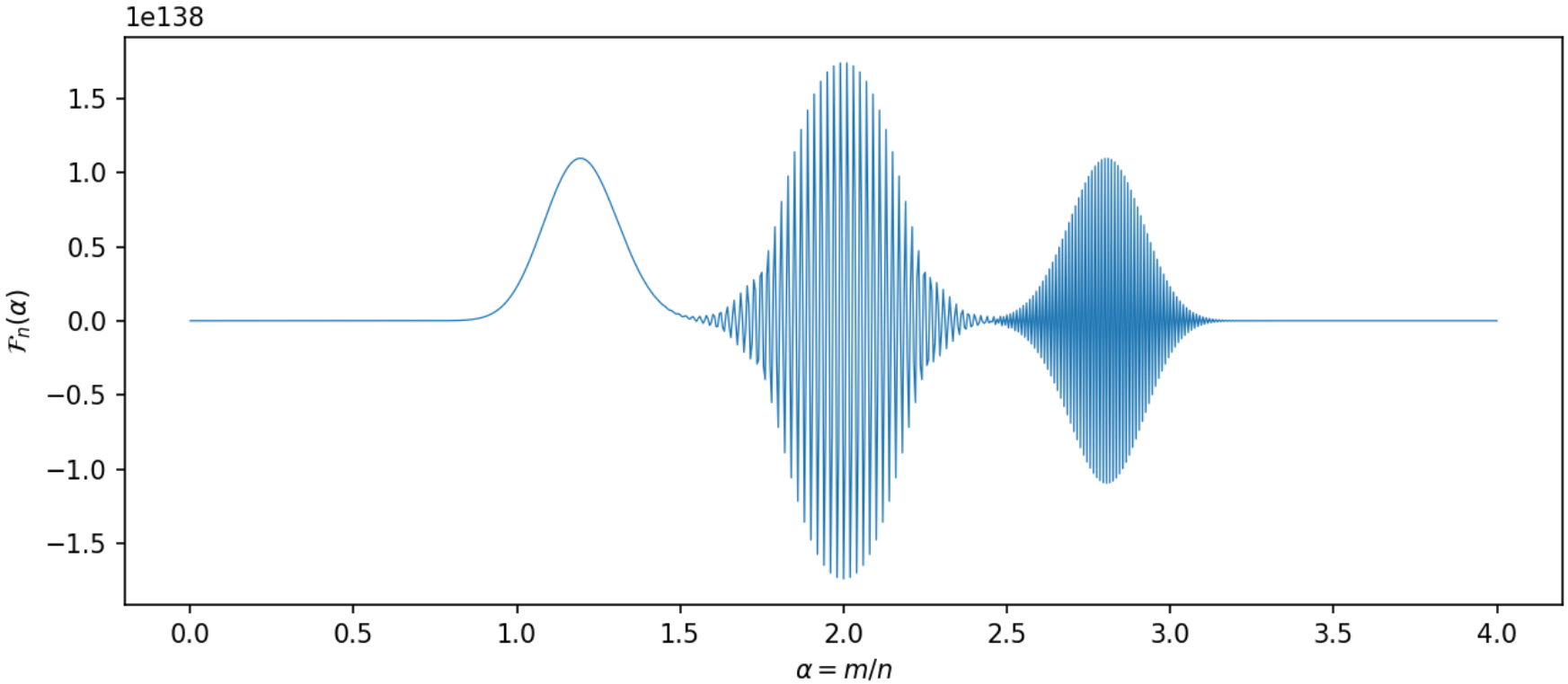


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Oscillations within Gaussian envelopes.

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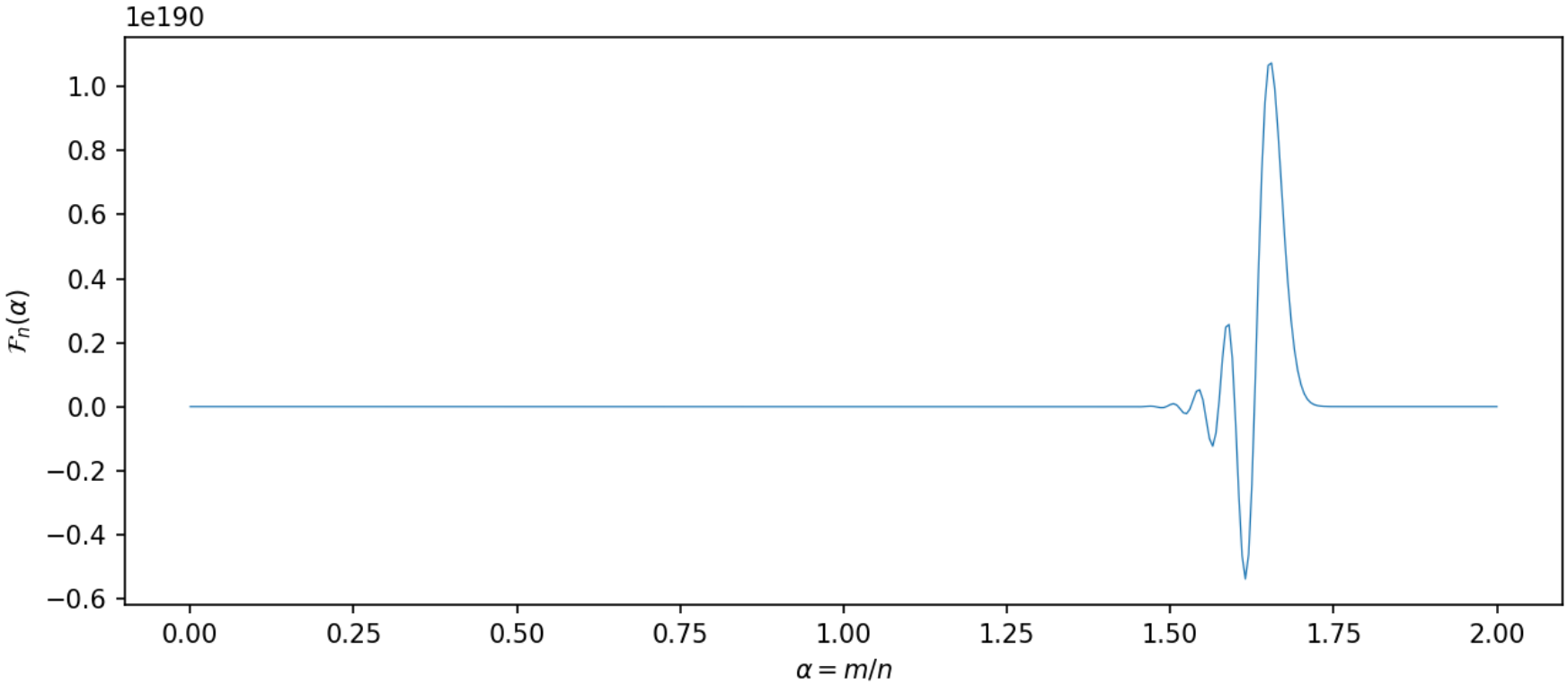


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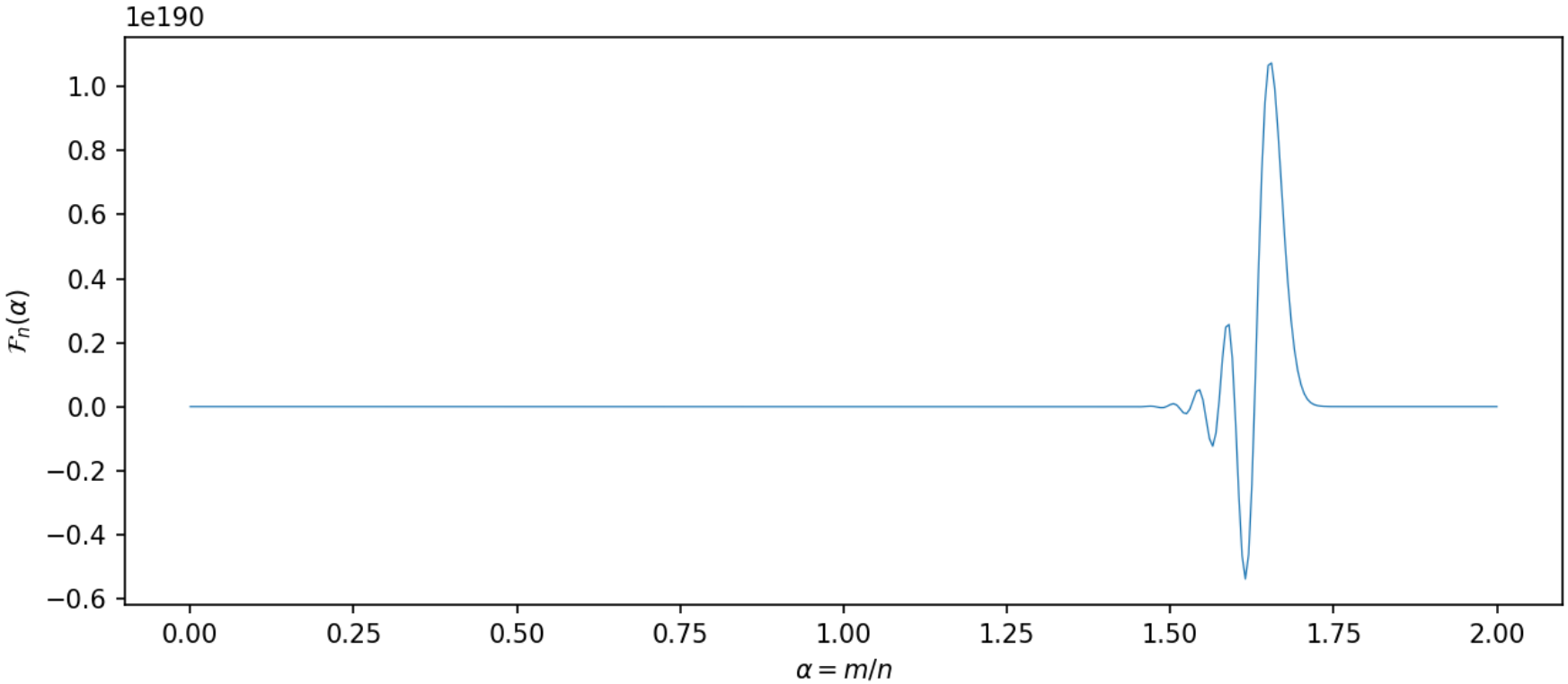


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Spoiler: Airy-type

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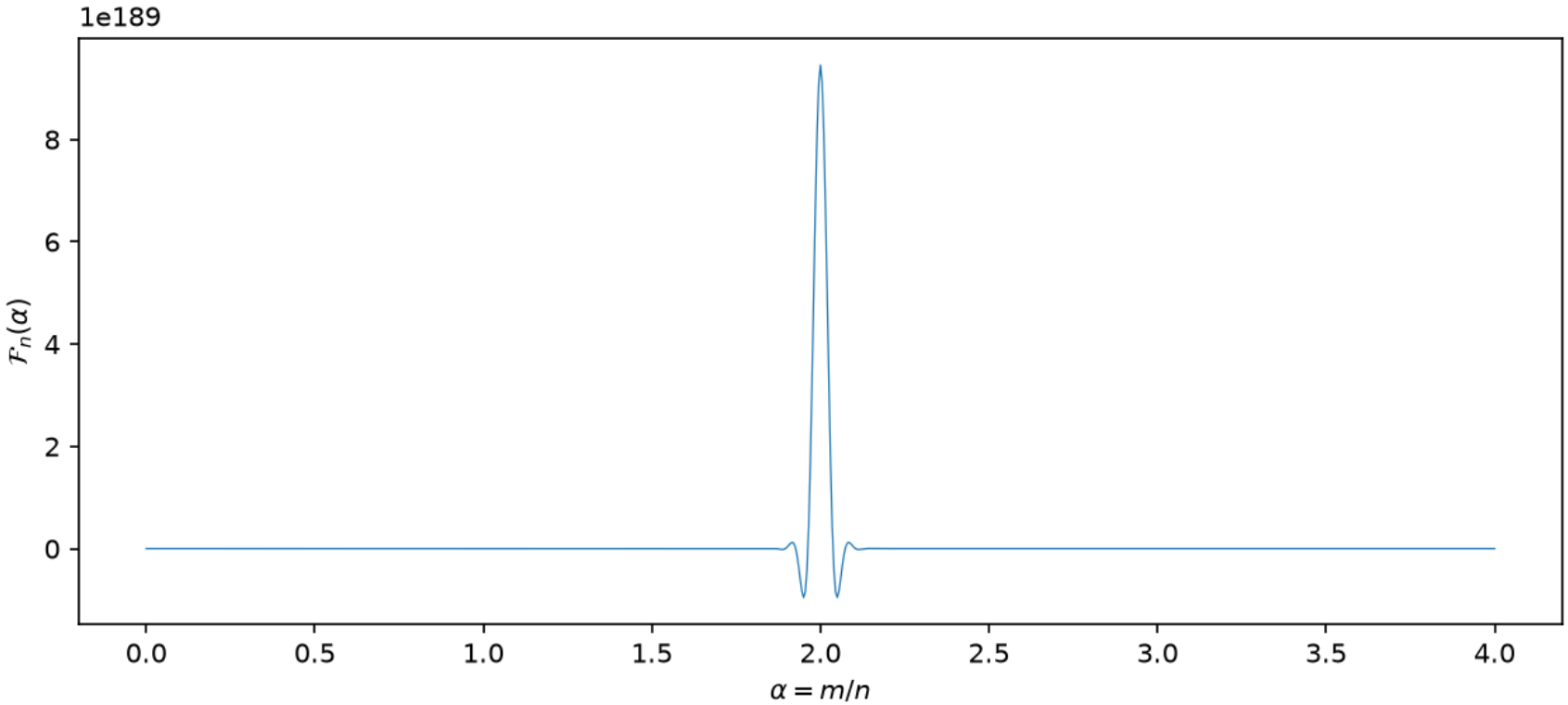


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A menagerie

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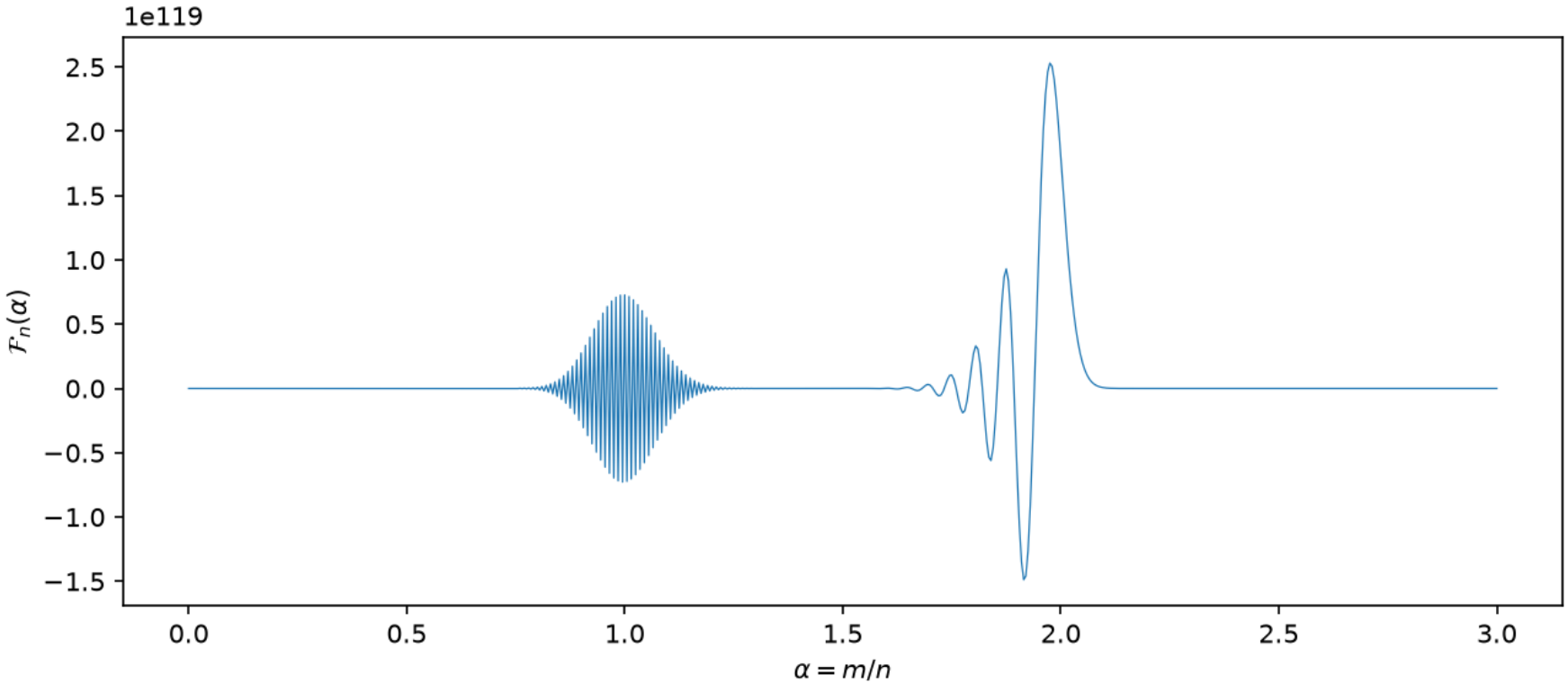


Figure: Coefficients $[x^m]P(x)^n$, with $n = 500$.

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$P(x) = 5 - 19x - 6x^2 - 14x^3 + x^4 + x^5$:

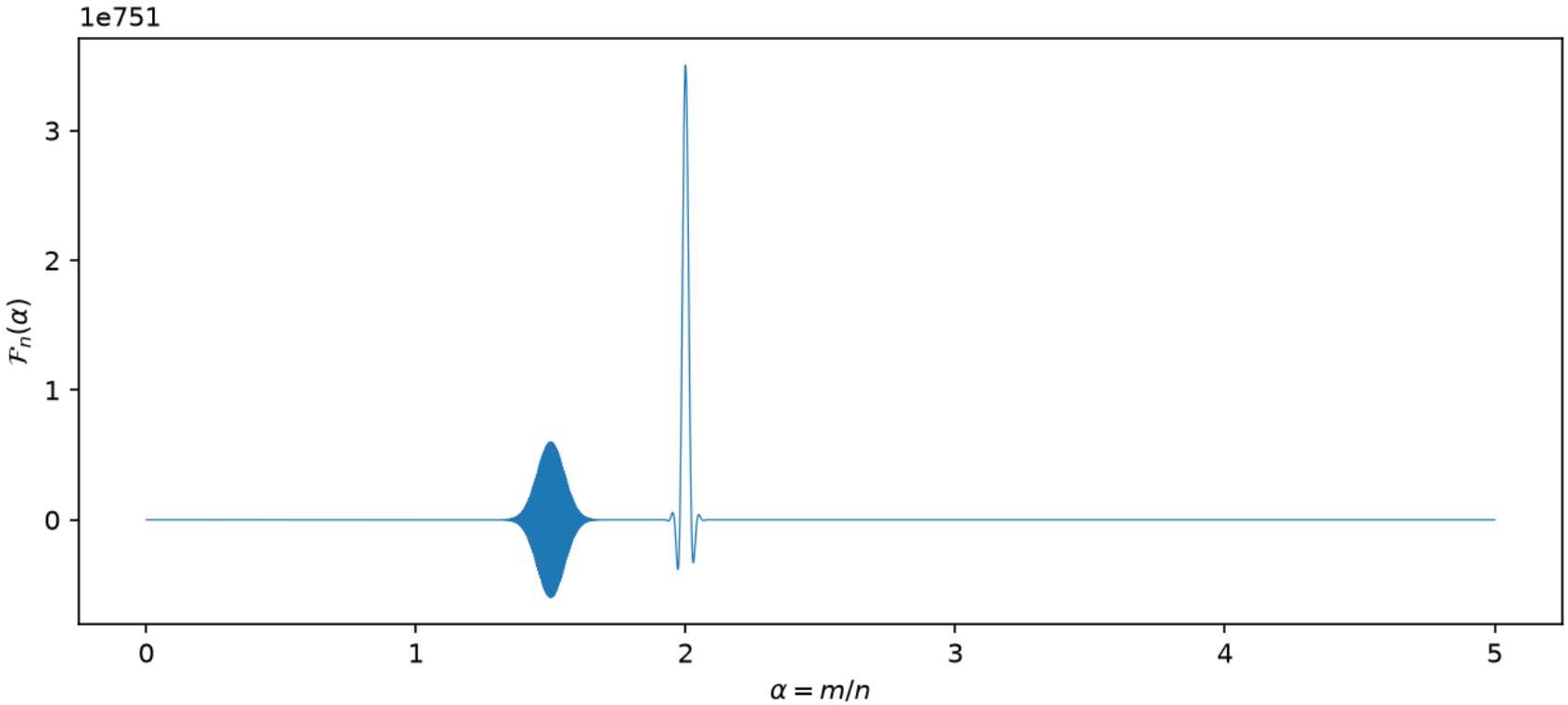


Figure: Coefficients $[x^m]P(x)^n$, with $n = 500$.

Note the Gaussian peak is wider.

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We will focus on intuition and examples over technical proofs.

Outline

1. Nonnegative Coefficients and Central Limit Theorem
2. Local Limit Theorems

Generating functions

Let $P(x) = a_0 + a_1 x + \cdots + a_k x^k$ have nonnegative coefficients.

$$Z \sim \begin{cases} \Pr(Z = 0) & = \frac{a_0}{P(1)} \\ \Pr(Z = 1) & = \frac{a_1}{P(1)} \\ \Pr(Z = 2) & = \frac{a_2}{P(1)} \\ & \vdots \end{cases}$$

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We want to know the distribution of $Z_1 + \cdots + Z_n$, since graphing $P(1)^n \Pr\left[\frac{Z_1+\cdots+Z_n}{n} = \alpha\right]$ against α yields $\mathcal{F}_{n,P}$.

Central Limit Theorem

Theorem (Central Limit Theorem)

Let $Z_1, Z_2, \ldots$ be i.i.d. copies of a scalar random variable Z of finite mean $\mu := \mathbb{E}[Z]$ and finite nonzero variance $\sigma^2 := \mathrm{Var}(Z)$. Let $S_n := Z_1 + \cdots + Z_n$. Then the random variables $\frac{\sqrt{n}}{\sigma}\left(\frac{S_n}{n} - \mu\right)$ converge in distribution to a random variable with the standard normal distribution $\mathcal{N}(0, 1)$.

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Then $\frac{Z_1+\cdots+Z_n}{n}$ is approximated by a normal distribution with mean μ and variance σ^2/n. In terms of P, we have

$$\mu = \frac{P'(1)}{P(1)}, \qquad \sigma^2 = \frac{P(1)P''(1) - P'(1)^2}{P(1)^2} + \frac{P'(1)}{P(1)}.$$

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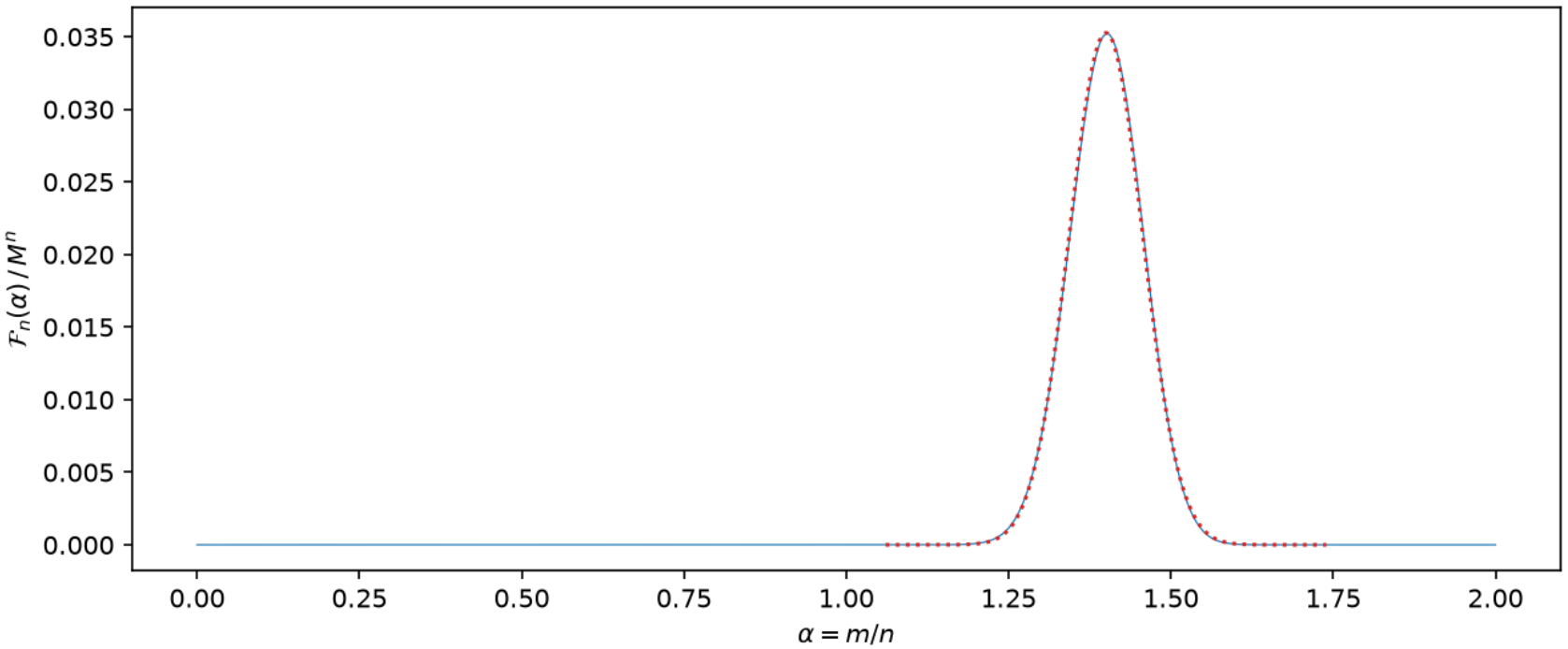


Figure: Plot of $\mathcal{F}_{n,P}(\alpha)/P(1)^n$ against the predicted Gaussian, $n = 100$.

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The Central Limit Theorem gives only convergence in distribution.

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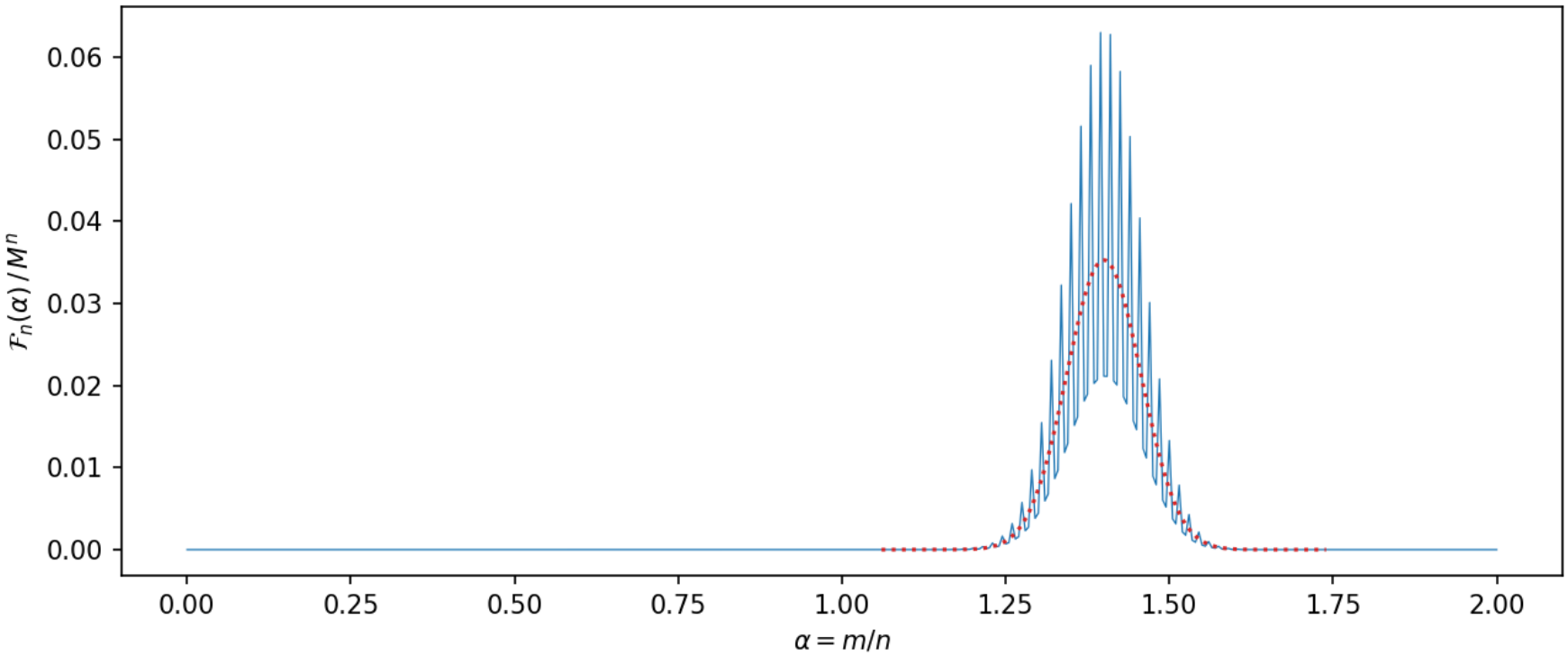


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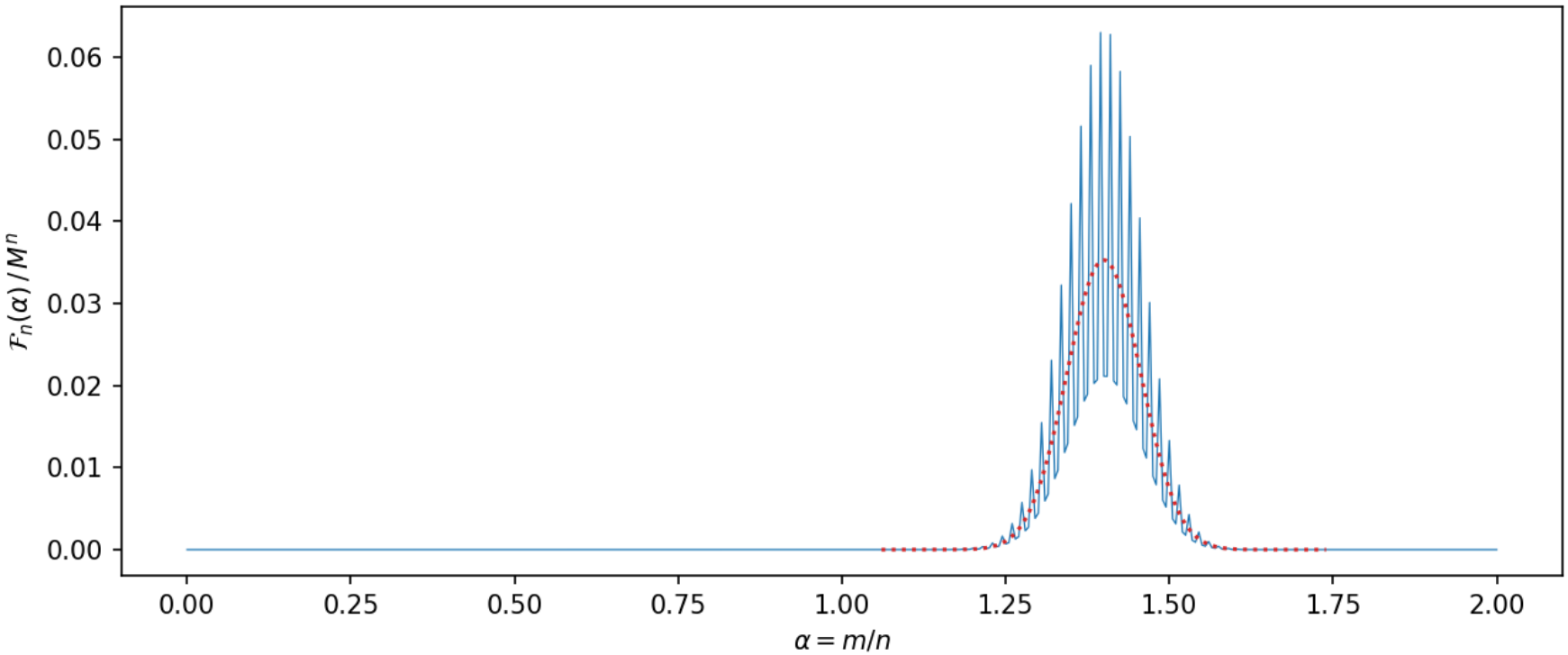


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We want a stronger pointwise convergence.

Outline

1 Nonnegative Coefficients and Central Limit Theorem

2 Local Limit Theorems

The Theorem (Gaussian Case)

Theorem

Let $P(x) \in \mathbb{R}[x]$ have nonnegative coefficients and not be a polynomial in x^d $(d \geq 2)$ after factoring out the lowest power. Then $|P(z)|$ achieves its maximum on the unit circle uniquely at $z = 1$. Let $\alpha_0 = \frac{P'(1)}{P(1)}$, and assume $c_2 = \frac{P(1)P''(1) - P'(1)^2}{P(1)^2} + \frac{P'(1)}{P(1)}$ is nonzero. Then for real μ,

$$\frac{1}{P(1)^n} \mathcal{F}_{n,P}(\alpha_0 + \mu n^{-1/2}) \sim \frac{1}{\sqrt{2\pi n c_2}} e^{-\mu^2/2c_2},$$

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Thus $\mathcal{F}_{n,P}$ converges pointwise to a Gaussian.

What about non-Gaussians?

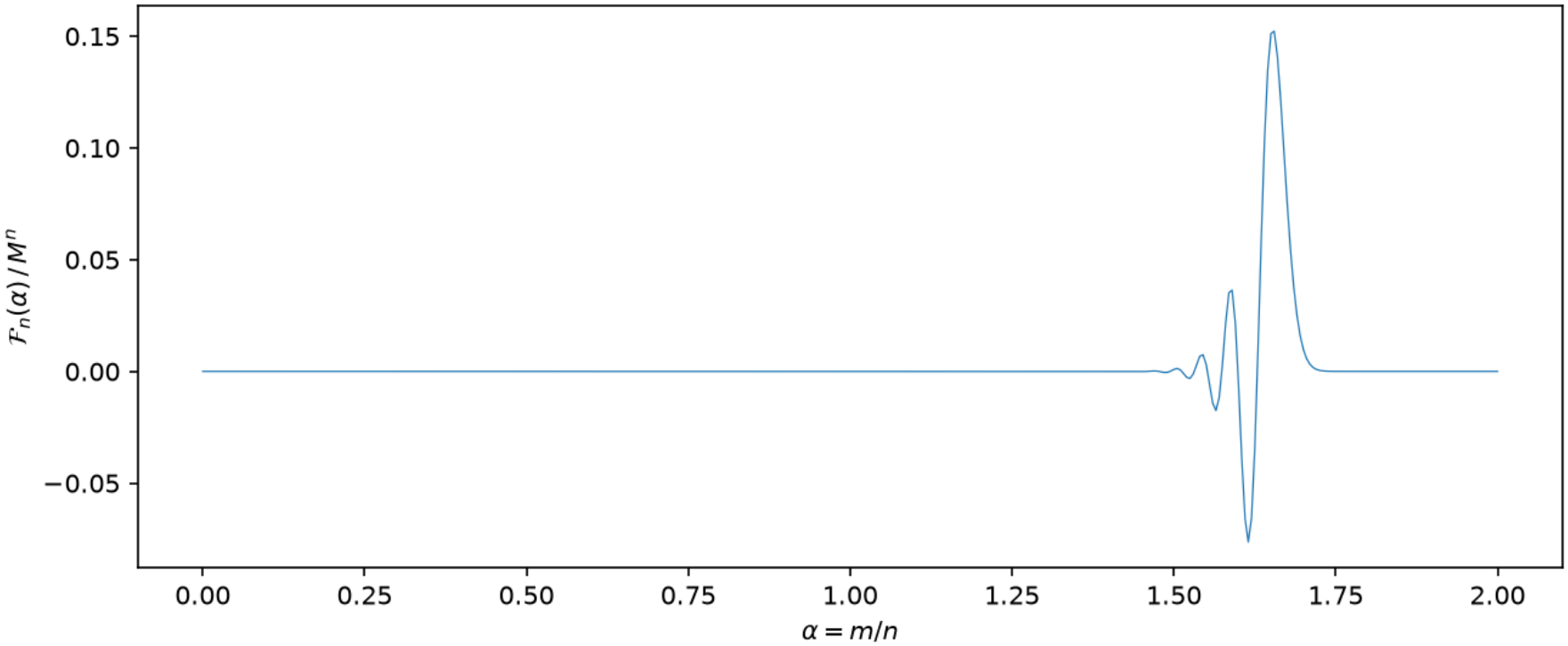


Figure: Coefficients $[x^m]P(x)^n$; $P(x) = -1 + 5x + 5x^2$, $n = 200$.

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Let $P(x) \in \mathbb{R}[x]$. Assume $|P(z)|$ has a unique maximum M on the unit circle, achieved at z_0. Let $\vartheta = z\frac{\mathrm{d}}{\mathrm{d}z}$, $\alpha_0 = \vartheta \log P(z_0)$, and c_q be the first nonzero value of $\vartheta^q \log P(z_0)$ for $q \geq 2$. Then for real μ, letting $e^{i\Phi} = z_0^{-\alpha} e^{i \arg P(z_0)}$, we have

$$\tfrac{1}{M^n}\mathcal{F}_{n,P}\left(\alpha_0 + \mu n^{-1+1/q}\right) \sim n^{-1/q}\frac{1}{2\pi}\int_{\mathbb{R}} \exp\left(-i\mu s + \tfrac{1}{q!}c_q i^q s^q\right)\mathrm{d}s \cdot e^{in\Phi}$$

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- When $z_0 = 1$ and $P(1) > 0$, the phase vanishes.

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Let $P(x) \in \mathbb{R}[x]$. Assume $|P(z)|$ has a unique maximum M on the unit circle, achieved at z_0. Let $\vartheta = z\frac{\mathrm{d}}{\mathrm{d}z}$, $\alpha_0 = \vartheta \log P(z_0)$, and c_q be the first nonzero value of $\vartheta^q \log P(z_0)$ for $q \geq 2$. Then for real μ, letting $e^{i\Phi} = z_0^{-\alpha} e^{i \arg P(z_0)}$, we have

$$\frac{1}{M^n}\mathcal{F}_{n,P}(\alpha_0 + \mu n^{-1+1/q}) \sim n^{-1/q}\frac{1}{2\pi}\int_{\mathbb{R}} \exp\left(-i\mu s + \tfrac{1}{q!}c_q i^q s^q\right)\mathrm{d}s \cdot e^{in\Phi}$$

away from the zeros of the real-valued function $A_q(\mu) = \int_{\mathbb{R}} \exp\left(-i\mu s + \frac{1}{q!}c_q i^q s^q\right)\mathrm{d}s$. If the maximum of $|P(z)|$ is not unique, then we may sum the RHS over the maxima with the same α_0.

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- On the graph, the width of a peak is of order $n^{-1+1/q}$.

Example: Airy

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$$\frac{1}{9^n}\mathcal{F}_{n,P}\left(\frac{5}{3} + \mu n^{-2/3}\right) \sim n^{-1/3}\frac{1}{2\pi}\int_{\mathbb{R}} \exp\left(-i\mu s - i\frac{5}{81}s^3\right) \mathrm{d}s\,,$$

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and

$$[x^m]P(x)^n \sim 9^n n^{-1/3}\frac{3}{\sqrt[3]{5}}\,\mathrm{Ai}\left(\frac{3}{\sqrt[3]{5}}(\alpha - \tfrac{5}{3})n^{2/3}\right)$$

near $\alpha = \frac{5}{3}$, where $\alpha = m/n$.

Example: Airy

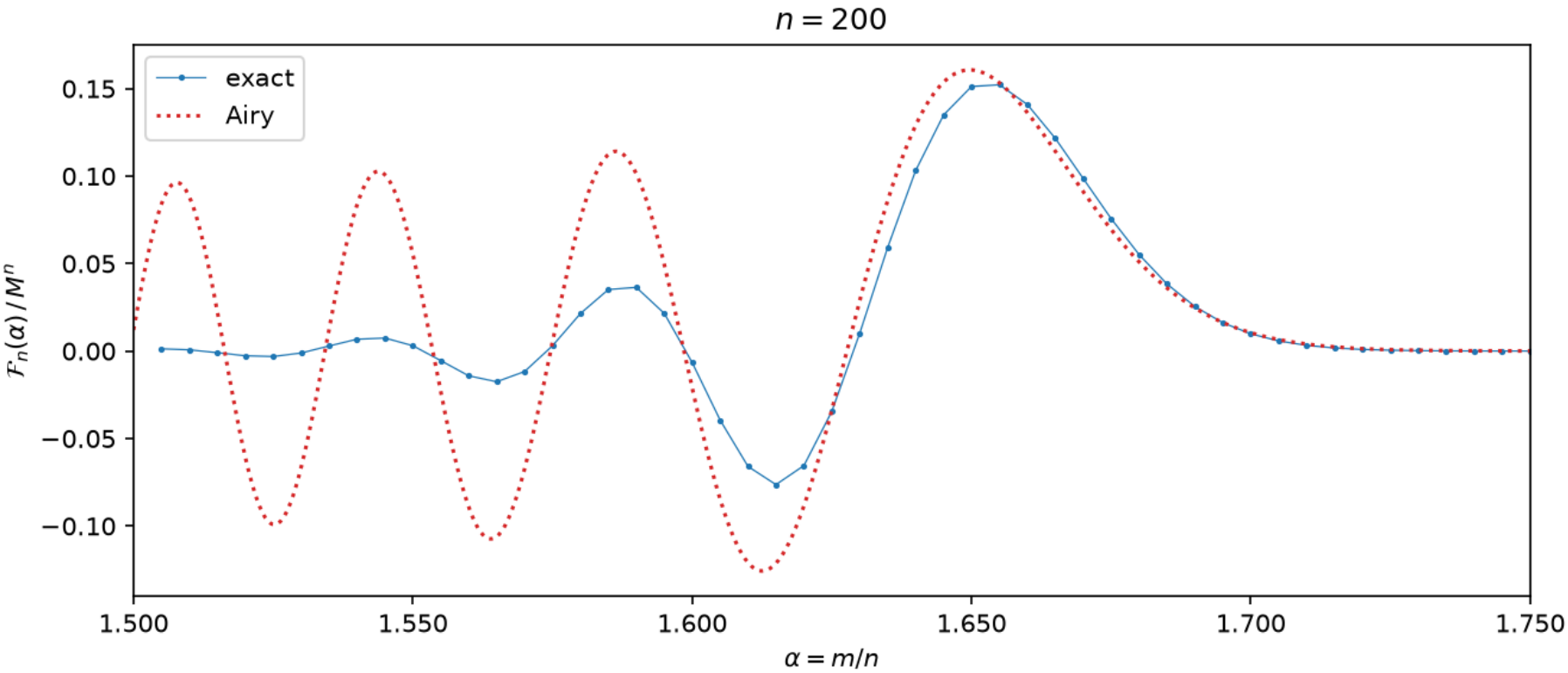


Figure: Coefficients $[x^m]P(x)^n$; $P(x) = -1 + 5x + 5x^2$ near $\alpha = 5/3$, $n = 200$.

Example: Airy

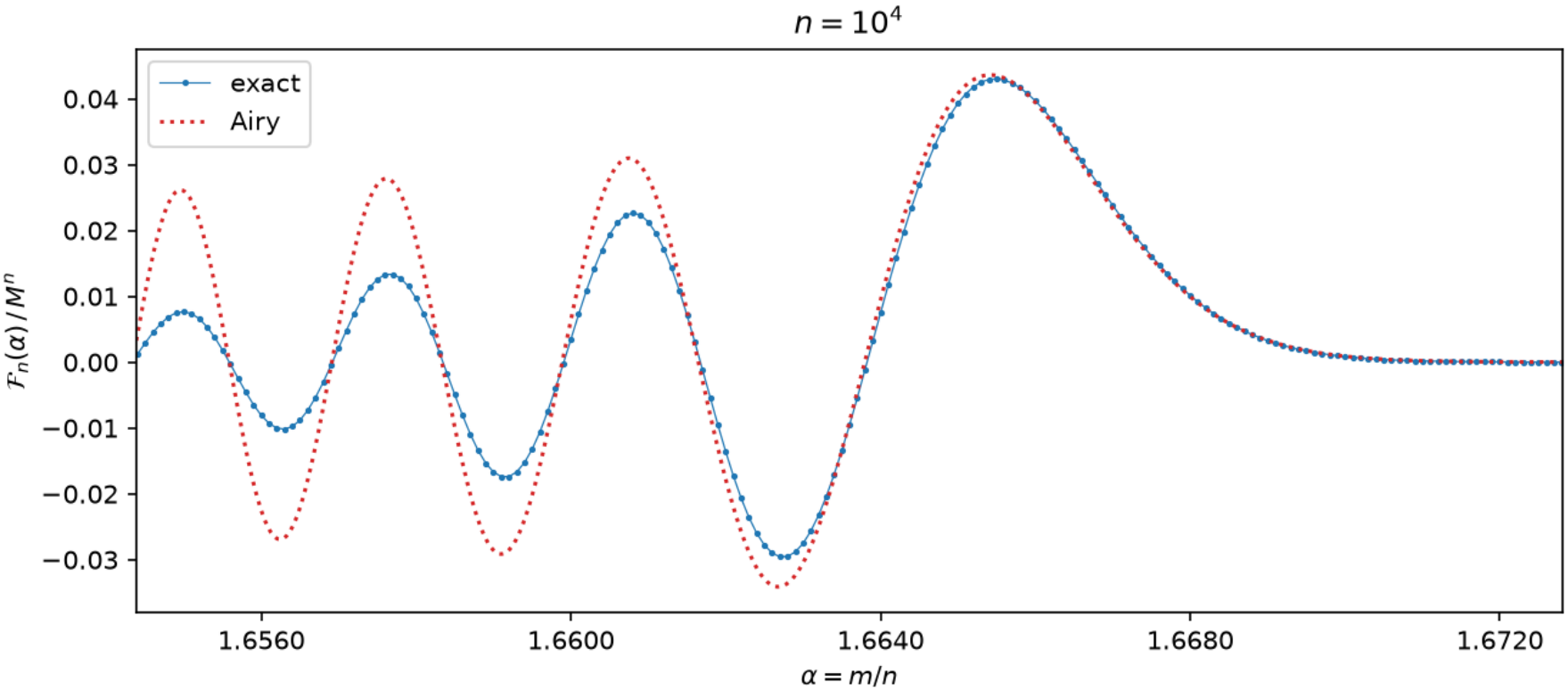


Figure: Coefficients $[x^m]P(x)^n$; $P(x) = -1 + 5x + 5x^2$ near $\alpha = 5/3$, $n = 10^4$.

Example: Airy

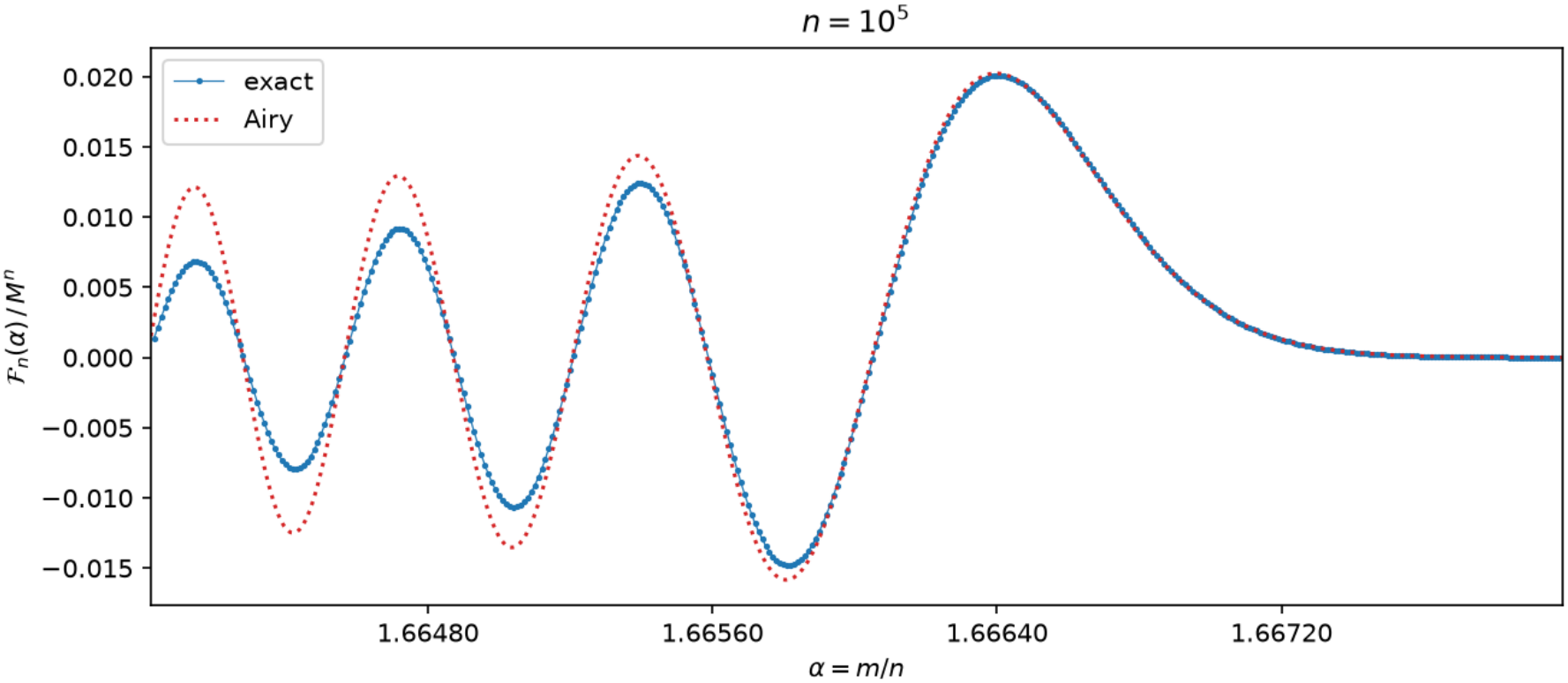


Figure: Coefficients $[x^m]P(x)^n$; $P(x) = -1 + 5x + 5x^2$ near $\alpha = 5/3$, $n = 10^5$.

Example: Airy

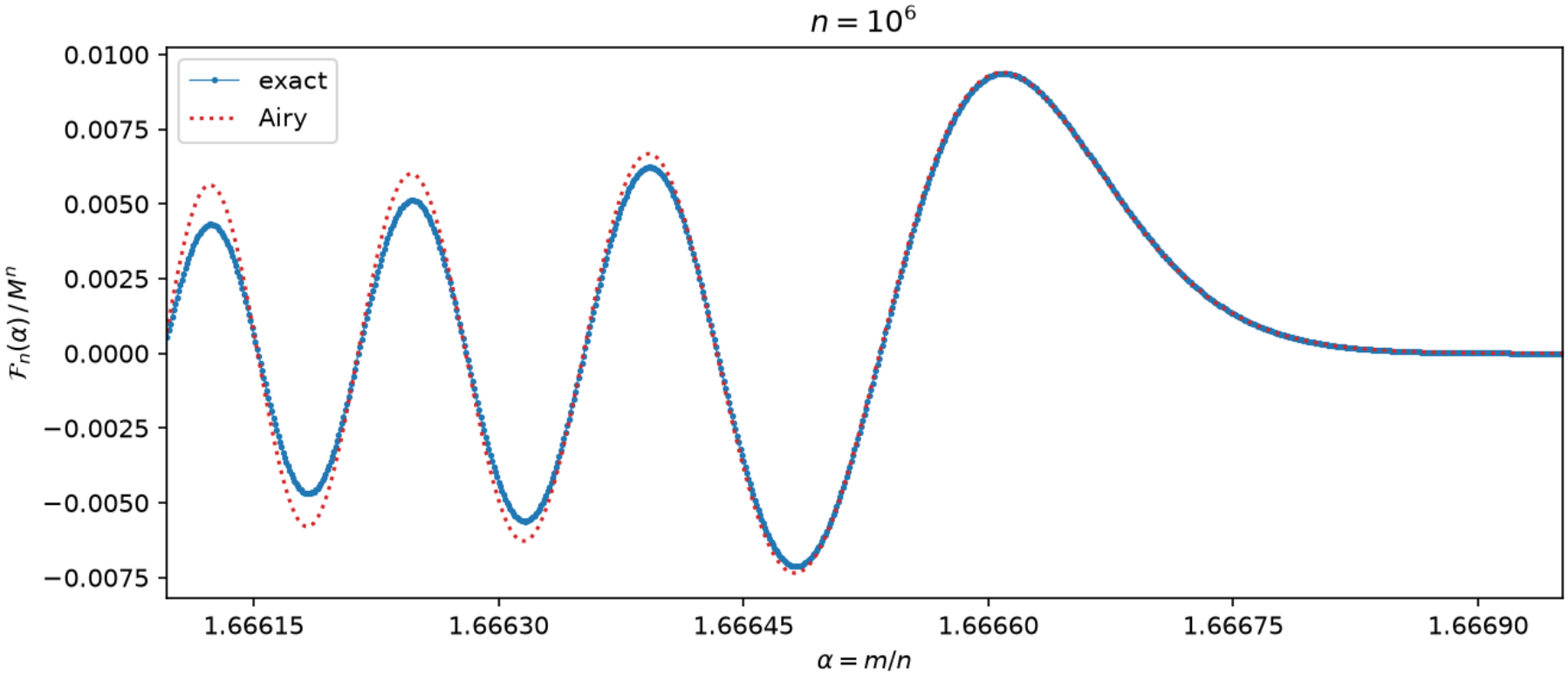


Figure: Coefficients $[x^m]P(x)^n$; $P(x) = -1 + 5x + 5x^2$ near $\alpha = 5/3$, $n = 10^6$.

Example: Airy

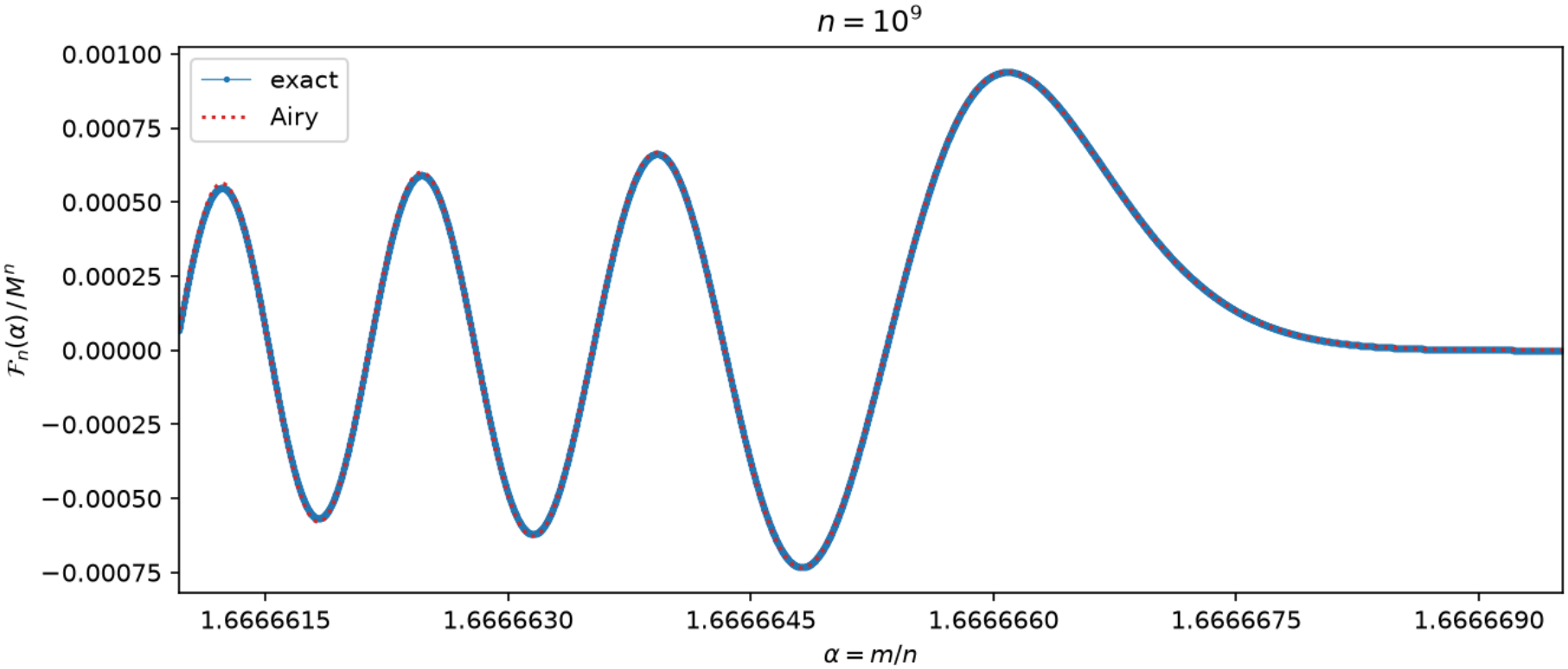


Figure: Coefficients $[x^m]P(x)^n$; $P(x) = -1 + 5x + 5x^2$ near $\alpha = 5/3$, $n = 10^9$.

Example: Airy

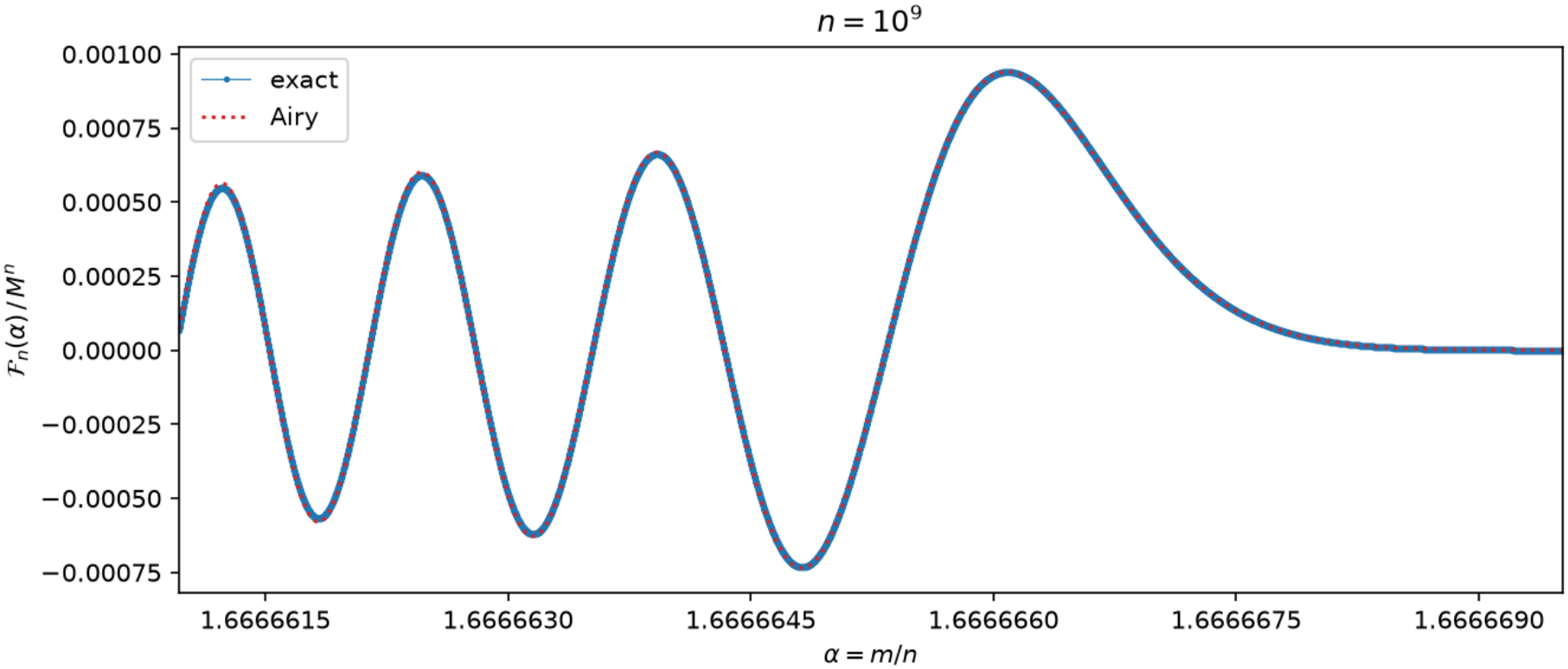


Figure: Coefficients $[x^m]P(x)^n$; $P(x) = -1 + 5x + 5x^2$ near $\alpha = 5/3$, $n = 10^9$.

Very nice :)

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Thank you for your attention!

Questions?