# Injectivity of Symmetric Polynomial Maps on Partitions

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# Elementary symmetric polynomial

The $k$-th elementary symmetric polynomial $e_k$ is the polynomial over $\ell \geq k$ variables defined by

$$e_k(x_1, x_2, \ldots, x_\ell) = \sum_{1 \leq i_1 < i_2 < \cdots < i_k \leq \ell} x_{i_1} x_{i_2} \ldots x_{i_k}$$

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## Example ($e_2$)

$e_2(x_1, x_2, x_3, x_4) = x_1x_2 + x_1x_3 + x_1x_4 + x_2x_3 + x_2x_4 + x_3x_4$

# What is $\mathrm{pre}_k$?

## Definition

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$$\begin{aligned} e_2(\lambda_1, \lambda_2, \lambda_3, \lambda_4) &= \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4 \\ &= 12 + 8 + 4 + 6 + 3 + 2 \end{aligned}$$

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$$\mathrm{pre}_2(\lambda) = (12, 8, 6, 4, 3, 2)$$

# Study of injectivity

We say $\mathrm{pre}_k$ is injective over $\mathcal{P}(n)$ if for any two partitions $\lambda, \mu$ of size $n$,

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Why study over $\mathcal{P}(n)$:

- $\lambda = (2,2) \in \mathcal{P}(4)$
- $\mu = (4,1) \in \mathcal{P}(5)$

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# Previous work on $\mathrm{pre}_k$

**Theorem (Li [2])**

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**Theorem (Hadelyn, Niergarth, Li, Li. [4])**

*For integers $n \geq k \geq 3$, the map $\mathrm{pre}_k$ fails to be injective over partitions of size $n$ with length $2k$.*

# Our work on $\mathrm{pre}_k$ I

## Theorem (T.–Tung)

*Let $\ell \geq k \geq 2$ and $\lambda, \mu \in \mathcal{P}_\ell(n)$. If $\lambda_i = \mu_i$ for $1 \leq i \leq k-1$ and $\mathrm{pre}_k(\lambda) = \mathrm{pre}_k(\mu)$, then $\lambda = \mu$.*

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## Example

If $k = 2$:

$$\lambda_1 = \mu_1$$

# Our work on $\mathrm{pre}_k$ II

**Theorem (T.–Tung)**

*For integers $m \geq k \geq 2$ and $n \geq 0$, the map $\mathrm{pre}_k$ is injective on the set of $m$-ary partitions of $n$ with at least $k$ parts.*

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- $3's = 2$
- $9's = 1$

Thus, $\lambda = (9, 3, 3, 1)$.

# Complete homogenous polynomials and $\mathrm{prh}_k$

The $k$-th complete homogenous symmetric polynomial $h_k$ is the polynomial over $\ell$ variables defined by:

$$h_k(x_1, x_2, \ldots, x_\ell) = \sum_{1 \le i_1 \le i_2 \le \cdots \le i_k \le \ell} x_{i_1} x_{i_2} \ldots x_{i_k}.$$

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## Example ($\mathrm{prh}_2$)

$$\mathrm{prh}_2(\lambda_1, \lambda_2) = (\lambda_1^2, \lambda_1\lambda_2, \lambda_2^2)$$

# Injectivity of $\mathrm{prh}$

We say $\mathrm{prh}_k$ is injective over $\mathcal{P}(n)$ if for any two partitions $\lambda, \mu$ of size $n$,

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**Theorem (T. – Tung)**

*For positive integers $k$, the map $\mathrm{prh}_k$ is injective over $\mathcal{P}$.*

# Young diagrams

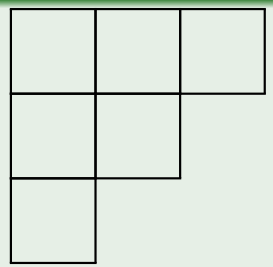
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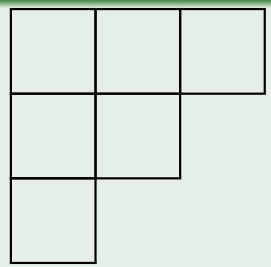


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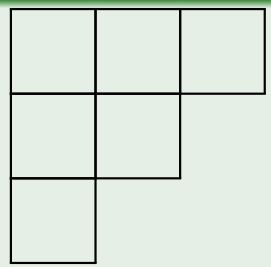
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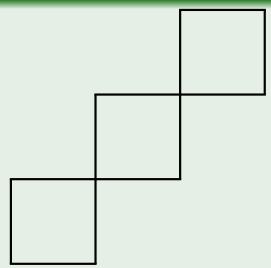
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## Example

$\lambda/\mu = (3, 2, 1)/(2, 1)$



# Semi-Standard Young Tableaux

A skew semi-standard Young tableau (SSYT) of shape $\lambda/\mu$ is a filling of the squares of $\lambda/\mu$ with positive integers such that

- the entries are weakly increasing along every row
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For a given SSYT $T$ and a set of variables $x = (x_1, x_2, \ldots, x_n)$, we let

$$x^T = x_1^{\#1's} x_2^{\#2's} \cdots x_n^{\#n's}.$$

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If $x = (x_1, x_2, x_3)$, then $x^T = x_1^2 x_2^2 x_3^2$.

# (Skew) Schur symmetric polynomial

The (skew) Schur polynomial $s_{\lambda/\mu}(x)$ where $x = (x_1, x_2, \ldots, x_\ell)$ is defined as

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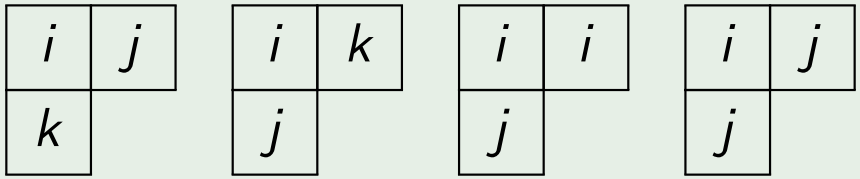
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$\lambda = (2,1)$, $\mu = \emptyset$, $x = (x_1, x_2, \ldots, x_n)$ for $n \geq 3$. $1 \leq i < j < k \leq n$



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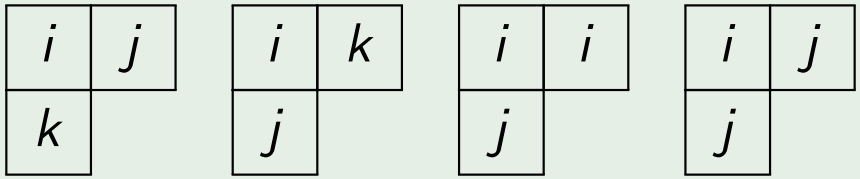
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Thus, it follows that

$$s_{(2,1)}(x_1, \ldots, x_n) = \sum_{1 \leq i < j \leq n} (x_i^2 x_j + x_i x_j^2) + 2 \sum_{1 \leq i < j < k \leq n} x_i x_j x_k.$$

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- $\mathrm{prs}_{(1^k)} = \mathrm{pre}_k$
- $\mathrm{prs}_{(k)} = \mathrm{prh}_k$

# Results on $\mathrm{prs}_\lambda$

## Theorem (T.– Tung)

*For all positive integers $n \geq 2$, the map $\mathrm{prs}_{(2,1)}$ is injective on the set of partitions of $n$ with at least two parts.*

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- $\lambda_1 = \lim_{a\to\infty}(\sum \lambda_i^a)^{1/a}$

# Acknowledgments

I would like to thank my mentor Katherine Tung for providing invaluable support and guidance throughout this whole research process. I would like to also thank the MIT PRIMES-USA program for giving me the opportunity to work on this project.

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# Future directions

For every two-row partition $\lambda'$ the function $\mathrm{prs}_{\lambda'}$ is injective on $\mathcal{P}(n)$?

$$s_{(2,1)} = \frac{1}{3}(p_1^3 - p_3)$$

$$s_{(1,1)} = e_2 = \frac{1}{2}(p_1^2 - p_2)$$

$$s_\lambda = \frac{1}{n}(p_1^n - p_n)?$$