

Program for Research in Mathematics, Engineering and Science



Photo: Slava Gerovitch

Fifteenth Annual Spring-Term PRIMES Conference May 18, 2025

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Sunday, May 18, 2025

Mathematics

9:45 am: Welcoming Remarks

- Prof. Pavel Etingof, PRIMES Chief Research Advisor
- Dr. Slava Gerovitch, PRIMES Program Director
- Marisa Gaetz and Mary Stelow, PRIMES Circle coordinators

10:00-10:45 am: Session 1: PRIMES Circle

- David Moon and Brian Huang, "Polyhedra and Euler Characteristics" (mentor Luis Modes)
- Brian Pan, Daniel Wang, and Nathan Nie, "An Introduction to Spectral Graph Theory" (mentor George Shaker)
- Salma Jama and Fantice Lin, "Donuts in Different Dimensions" (mentor Ezra Guerrero)

10:55-11:55 am: Session 2: PRIMES Circle

- Auden Nash and Daniil Sheenko, "Periodicity of S-Pick Up Bricks" (mentor Honglin Zhu)
- Boston Bulis, Kyra Burke, and Lee Van Voorhis, "Number Theory" (mentor Sam Packman)
- Joseph Kasongo and Jordan Lin, "Why Your Friends are More Popular than You (Statistically Speaking)" (mentor Kai Edick)
- Marko Mano, Kalle Nikula-Gill, and Derek Yin, "Dynamical Systems and Chaos Theory" (mentor Sean Li)

12:05-1:00 pm: Session 3: Math Research and PRIMES STEP

- Evin Liang, "Cubics in Euclidean Geometry" (mentor Dr. Tanya Khovanova)
- Dillan Agrawal, Selena Ge, Jate Greene, Dohun Kim, Rajarshi Mandal, Tanish Parida, Anirudh Pulugurtha, Gordon Redwine, Soham Samanta, and Albert Xu

(PRIMES STEP Senior group), "Chip-Firing on Infinite *k*-ary Trees" (mentor Dr. Tanya Khovanova)

 Eric Huang, Timur Kilybayev, Ryan Li, Brandon Ni, Leone Seidel, Samarth Sharma, Vivek Varanasi, Alice Sien Yin, Boya Yun, William Zelevinsky (PRIMES STEP Junior group), "Playing Your Cards Right: The Art of Josephus Dealing" (mentor Dr. Tanya Khovanova)

2:00-3:00 pm: Session 4: PRIMES Circle

- Sage Kramer, Runxuan Lin, and Anna Lu, "Set Theory and Logic" (mentor Zoe Xi)
- Leena Bhandari Cordoba and Sofia Bhandari Cordoba, "Number Theory: the Difference Game" (mentor Amy He)
- Celina Hwang, Lena Lee, and Emma Liu, "Algorithmic Exploration in Graph Theory" (mentor Julia Kozak)
- Jianing Huang and Sylvia Lee, "Introduction to Group and Ring Theory: The Foundations of Algebraic Structures" (mentor June Kayath)

3:10-3:55 pm: Session 5: PRIMES Circle

- Hannah Ahn and Carolena Douglas, "Pick's Theorem: How to Calculate the Area of a Polygon" (mentor Katherine Tung)
- Shamini Biju, Sherri Wu, and ZZ Zhang, "Understanding the Basics of the Theory of Computation" (mentor Katherine Taylor)
- Penelope Newsome and Amarachi Okeke, "Knot Theory" (mentor Sabine Chu)

Computer Science

4:15-5:00 pm: Session 6: Computer Science Research

- Coleman DuPlessie, "Characterizing Interpretable Features of Reinforcement Learning Models" (mentor Andrew Gritsevskiy, University of Wisconsin-Madison)
- Shreyas Ekanathan, "A Fully Adaptive Radau Method to Efficiently Solve Stiff
 Ordinary Differential Equations" (mentor Dr. Chris Rackauckas)
- Govind Velamoor, "System-wide Timeout Strategies" (mentor Prof. Raja Sambasivan, Tufts University)

2025 PRIMES SPRING CONFERENCE ABSTRACTS

SESSION 1. PRIMES CIRCLE

Brian Huang & David Moon

Polyhedra and Euler Characteristics

Mentor: Luis Modes

In this presentation, we cover the traits of polyhedra and introduce topology. For polyhedra, we prove the existence of limited Platonic solids and introduce Euler's formula. Later, we define the Euler characteristic to explain why Euler's formula works. Regarding topology, we will cover \mathbb{RP}^2 and cellular decomposition.

Nathan Nie, Brian Pan, & Daniel Wang

An Introduction to Spectral Graph Theory

Mentor: George Shaker

Spectral graph theory bridges the world of linear algebra and graph theory by studying how a graph's connectivity and organization are encoded in the eigenvalues and eigenvectors of its associated matrices. We begin with a concise overview of the fundamental concepts of graph theory and linear algebra. Then, we introduce the principal matrix representations of a graph: the adjacency matrix and incidence matrix. We highlight their definitions and properties. Next, we introduce the Laplacian matrix. Finally, we investigate the eigenvalues of the Laplacian matrix and the deep insights they reveal into the connectivity of the graph.

Salma Jama & Fantice Lin

Donuts in Different Dimensions

Mentor: Ezra Guerrero

What happens when you take a donut and change the number of dimensions it lives in? In this presentation, we'll use a donut—also known as a torus—as our guide through the world of dimensions. We'll start with simple 0D, 1D, and 2D shapes, then move into what it means for a shape to be 3-dimensional. From there, we'll explore how mathematicians study these shapes using tools like *isotopy*, which help us understand when shapes are "the same" even if they look different. All along the way, the donut will help us make sense of these abstract ideas.

Session 2. PRIMES CIRCLE

Auden Nash & Daniil Sheenko

Periodicity of S-Pick-Up-Bricks

Mentor: Honglin Zhu

We provide an intuitive definition of positions and state that every position in an impartial game is equivalent to a nimber. We state the MEX principle, an approach allowing us to find the nimber any position is equivalent to. Lastly, we introduce the game of S-Pick-Up-Bricks and demonstrate periodicity in the sequence of nimbers of its game positions.

Boston Bulis, Kyra Burke, & Lee Van Voorhis

Number Theory: Why MIT PRIMES Circle Could Potentially be Renamed MIT PRIMES Triangle

Mentor: Sam Packman

Have you ever wondered what it really means to say two numbers are "congruent modulo n"? Or why operations as simple as addition and rotation can belong to the same mathematical framework? This presentation introduces several foundational ideas in number theory, beginning with modular arithmetic—a powerful tool for working with remainders. We then explore the basics of group theory, using examples to highlight key ideas and show how they connect back to modular systems. To bring these concepts to life, we take a closer look at the dihedral group of a triangle, a concrete and visually intuitive example. Along the way, we'll uncover how these areas overlap, offering a deeper appreciation for the structure and patterns within number theory.

Joseph Kasongo & Jordan Lin

Why Your Friends are More Popular than You (Statistically Speaking)

Mentor: Kai Edick

Probability theory began as a tool for analyzing games of chance in the 17th century, with foundational work by Blaise Pascal and Pierre de Fermat. Over time, it evolved into a powerful framework for understanding uncertainty across disciplines—from physics to social science. As the theory matured, it revealed that even simple averages can produce surprisingly counterintuitive results. One such example is the friendship paradox. Here's a fun fact: your friends probably have more friends than you. This presentation explores the mathematics behind that claim and shows how the paradox arises not from social failure, but from the logical structure of probability and sampling. Through this lens, we uncover how mathematical reasoning can explain everyday illusions.

Marko Mano, Kalle Nikula-Gill, & Derek Yin

Dynamical Systems and Chaos Theory

Mentor: Sean Li

This presentation will extrapolate upon the fundamental principles of dynamical systems and chaos theory—more precisely, discrete dynamical systems. The essential concept behind dynamical systems is the iteration of a function and investigating the function's chaotic or periodic behavior over time. Additionally, the preliminary concepts of defining fixed points and periodic points will be discussed. The focal point of the study is the quadratic family, whose behavior is analyzed as the parameter c (a constant that vertically shifts the function) varies, leading to bifurcations that signal profound qualitative changes in dynamics. Furthermore, we introduce the notion of attracting, repelling, and neutral fixed points via derivative criteria, and develop a bifurcation diagram that encapsulates these transitions. We will explore quadratic functions in the real number plane and further discuss the existence of fixed points and their behavior (attracting, repelling, or neutral). The understanding of quadratic functions will be applied from the real number plane to the complex plane. The transition from real to complex dimensions allows the extension from 1D to 2D space through Julia sets and the Mandelbrot set. We will explore these sets, their construction, and some of their properties.

Session 3. Math Research and PRIMES STEP

Evin Liang

Cubics in Euclidean Geometry

Mentor: Dr. Tanya Khovanova

Have you ever seen a curve that loops and twists in surprising ways? These are called cubics, and they appear all over geometry. A classical result called Pappus's theorem turns out to be just one part of a bigger picture involving cubics, thanks to Chasles's theorem. In this talk, we'll explore how cubics connect classical theorems with lots of colorful pictures. We'll end by looking at some special cubics that show up in triangle geometry.

Dillan Agrawal, Selena Ge, Jate Greene, Dohun Kim, Rajarshi Mandal, Tanish Parida, Anirudh Pulugurtha, Gordon Redwine, Soham Samanta, and Albert Xu (PRIMES STEP Senior group)

Chip-Firing on Infinite k-ary Trees

Mentor: Dr. Tanya Khovanova

We use an infinite k-ary tree with a self-loop at the root as our underlying graph and consider a chip-firing process starting with N chips at the root. One of our goals is to describe the stable configurations and calculate the number of fires for each vertex and the total number of fires. We also study a sequence of the number of root fires for a given k as a function of N and study its properties. We do the same for the total number of fires.

Eric Huang, Timur Kilybayev, Ryan Li, Brandon Ni, Leone Seidel, Samarth Sharma, Vivek Varanasi, Alice Sien Yin, Boya Yun, William Zelevinsky (PRIMES STEP Junior group)

Playing Your Cards Right: The Art of Josephus Dealing

Mentor: Dr. Tanya Khovanova

This presentation explores the connection between the Josephus problem and card dealing. We'll explain how the elimination process in the Josephus problem mirrors the order of dealing cards, using live demonstrations and a brief magic trick to illustrate key concepts. The talk will also highlight related integer sequences and provide an overview of relevant mathematical research.

SESSION 4. PRIMES CIRCLE

Sage Kramer, Runxuan Lin, & Anna Lu

Set Theory and Logic

Mentor: Zoe Xi

This presentation explores foundational concepts in mathematical logic and their real-world applications. We begin with sentential logic, focusing on constructing and interpreting truth tables to evaluate the validity of logical arguments. Building on this, we introduce set theory, emphasizing key operations and notations and construct a proof using proof by contradiction, a technique used to establish the truth of mathematical statements through logical negation. Finally, we demonstrate how these abstract ideas are used in the popular logic-based puzzle Sudoku. By analyzing Sudoku through the lens of logic and set theory, we discuss the hidden structure and constraints that make the game both challenging and solvable. Through a combination of theory and application, our

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goal is to illustrate and contextualize mathematical logic, reasoning, and problem-solving in both academic and real world contexts.

Leena Bhandari Cordoba & Sofia Bhandari Cordoba

Number Theory: The Difference Game

Mentor: Amy He

This presentation explores the Difference Game—a game which always has a fixed outcome. During the Difference Game, two players compete to be the last one to create a unique difference when given distinct positive numbers. We will also explore concepts such as the Well Ordering Principle, the division algorithm, and the greatest common divisor, as all three work to prove the predetermined termination of the Difference Game.

Celina Hwang, Lena Lee, & Emma Liu

Algorithmic Exploration in Graph Theory

Mentor: Julia Kozak

This presentation explores the fundamentals of graph theory and its role in the development of algorithms essential in today's interconnected world. We begin with foundational concepts in graph theory, including connectivity, chromatic numbers, and spanning trees, and present applications of graphs to real-world problems such as airline route optimization. This exploration underscores the importance of graph theory in understanding advanced modern computation systems.

Jianing Huang & Sylvia Lee

Introduction to Group and Ring Theory: The Foundations of Algebraic Structures

Mentor: June Kayath

This presentation aims to offer an introduction to some fundamental concepts of group theory, which is a central part of abstract algebra, or the study of algebraic structures. Beginning with the definition and properties of groups, illustrated by examples involving symmetries, number systems, and modular arithmetic, we then proceed to introduce subgroups—substructures within a group. The presentation concludes with a focus on cyclic groups, a type of group with elegant structures and generated by one element, by offering proof of the fundamental theorem of cyclic groups. The theorem explores the behavior of subgroups within cyclic groups and lays the basis for understanding more complex group behavior and properties.

Session 5. PRIMES Circle

Hannah Ahn & Carolena Douglas

Pick's Theorem: How to Calculate the Area of a Polygon

Mentor: Katherine Tung

We provide examples of various ways to calculate the area of a triangle, including Pick's theorem, first proved by Georg Alexander Pick in 1899. The theorem gives a formula for calculating the area of a simple polygon with lattice points as vertices. We give an example as to why the lattice point hypothesis is necessary, as well as a generalization of Pick's theorem for polygons with holes.

Shamini Biju, Sherri Wu, & ZZ Zhang

Understanding the Basics of the Theory of Computation

Mentor: Katherine Taylor

Our presentation will introduce deterministic finite automata and nondeterministic finite automata alongside languages and their use in the theory of computation and the real world. First, the definition of language and the phrase "recognized by a certain machine" will be presented to provide context on why machines are built to process strings. Then, we will dive straight into the simplest machines: Finite Automata, which include deterministic and nondeterministic finite automata. For each of these machines, we'll provide the formal definition and an example derived from Michael Sipser's book, *Introduction to the Theory of Computation*. Additionally, we will go over the equivalence between deterministic finite automata and nondeterministic finite automata, and how equivalence means that they recognize the same class of languages: regular languages. We will then talk about the real-world applications and relevance of this topic. Lastly, we will introduce depth-first search and breadth-first search and will discuss how they can be used to better understand nondeterminism.

Penelope Newsome & Amarachi Okeke

Knot Theory

Mentor: Sabine Chu

This presentation explores the role of surfaces in knot theory, emphasizing their significance both within and beyond the field. We'll discuss the foundational concepts of knot theory, the importance of surfaces, and how they help in distinguishing knots. This presentation also delves lightly into the construction and properties of Seifert surfaces, providing a comprehensive understanding of their applications.

Session 6. Computer Science Research

Coleman DuPlessie

Characterizing Interpretable Features of Reinforcement Learning Models

Mentor: Andrew Gritsevskiy, University of Wisconsin-Madison

Sparse autoencoders (SAEs) are machine learning models that can be used to express the inner workings of certain other models as human-interpretable features. While sparse autoencoders work well when applied to language models, there has been little research that investigates the extent to which they generalize to other applications of machine learning. This work investigates the application of SAEs to a deep Q network trained to complete a simple task, and finds that, although SAEs tend to perform well and find a number of human-interpretable features, they contain a large number of "dead features" that never activate, which suggests that more research is necessary to adapt SAEs to the unique tasks reinforcement learning models solve. In particular, we note that the most effective deep Q networks trained to complete a task tend to result in sparse autoencoders with a consistent number of dead features, suggesting that these sparse autoencoders may in some sense be capturing the "optimal" or "true" number of features needed to solve the toy problem we study, and the high quantity of dead features may simply imply that additional live features past a certain quantity are unhelpful.

Shreyas Ekanathan

A Fully Adaptive Radau Method to Efficiently Solve Stiff Ordinary Differential Equations

Mentor: Dr. Chris Rackauckas

Radau methods are known to be state-of-the-art for the high-accuracy solution of highly stiff ordinary differential equations (ODEs). However, the traditional implementation was specialized to a specific range of precision and only derived in floating point, thus limiting the algorithm's ability to be truly general purpose for highly accurate scenarios. To alleviate these constraints, we implemented a fully adaptive Radau method that can derive the coefficients for the Radau method on the fly to any precision. Additionally, the Julia-based implementation includes many modernizations to improve performance, including improved linear algebra integrations and parallelized computation. In a head-to-head benchmark against the classic Fortran implementation, we demonstrate our implementation is approximately 2x faster across a range of stiff ODEs. We further benchmark our algorithm against several well-reputed numerical integrators for stiff ODEs and find state-of-the-art performance on several test problems, with a 1.5x speed-up over common numerical integrators for stiff ODEs when high precision is required. In this talk, we describe the overall scheme of numerical integrators before diving into some of the intricacies of Radau methods, including the beauty of utilizing of the complex plane to accelerate computations and our adaptive time-stepping and order-selection scheme, which has helped us accelerate computation.

Govind Velamoor

System-wide Timeout Strategies

Mentor: Prof. Raja Sambasivan, Tufts University

Setting dynamic, efficient timeouts in distributed systems is crucial for reliability yet remains challenging due to the need for rapid adjustment and the risk of redundant computation. We address this by extending ALTO—our adaptive timeout optimization framework—into a systemwide solution via a Service Mesh–style deployment, enabling each service instance to coordinate timeout decisions using shared telemetry while incurring minimal overhead. We will first compare this global approach against the original per-service ALTO on a custom microservice testbed, measuring the number of timed out requests as an indicator of failure resilience, and subsequently evaluate at scale on SocialNetwork, an industry-standard benchmark, to validate scalability and guide real-world deployment.