Unveiling the High-Resolution Structure of Saturn's Rings from Cassini Radio Science Occultations

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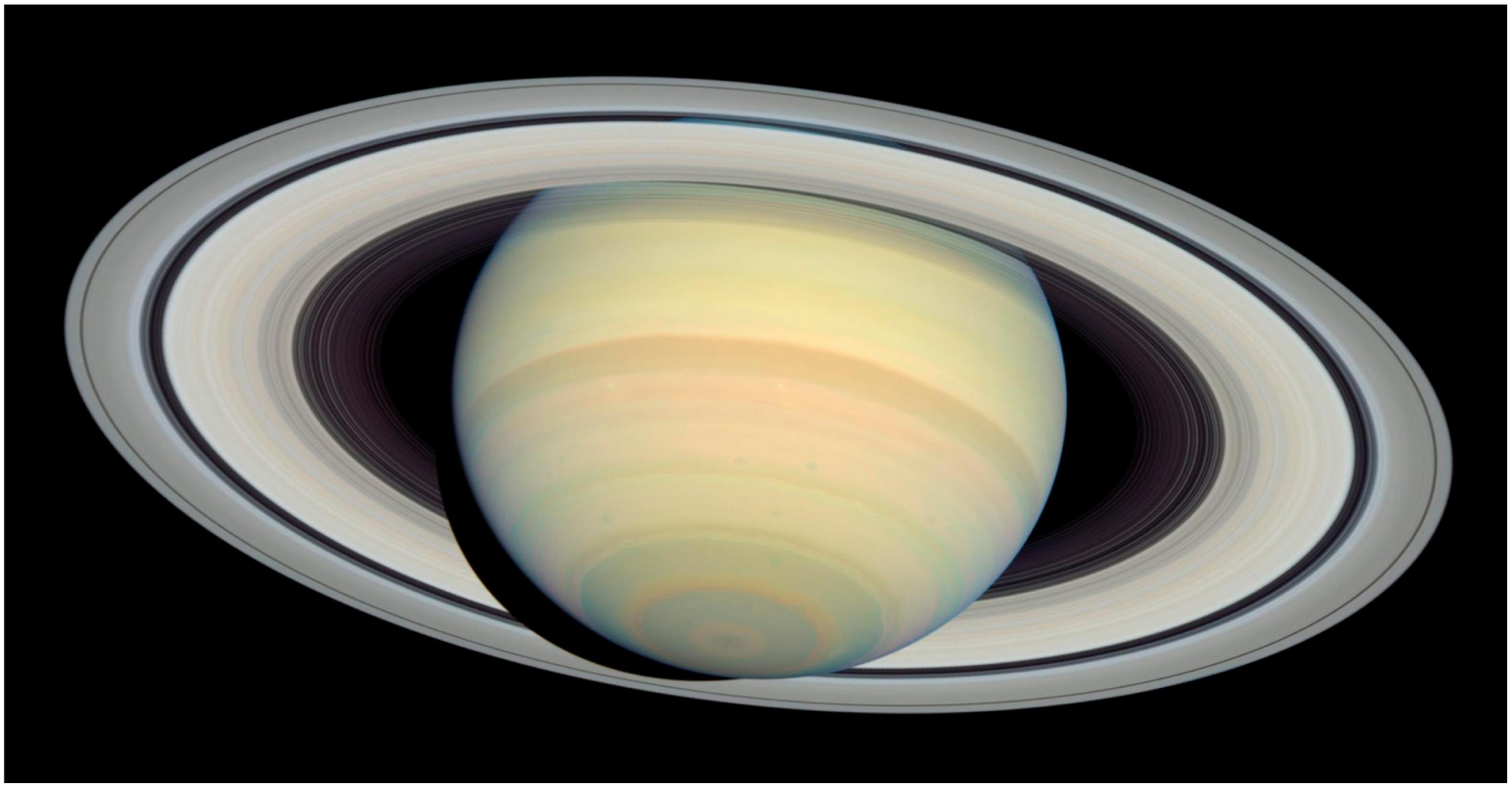
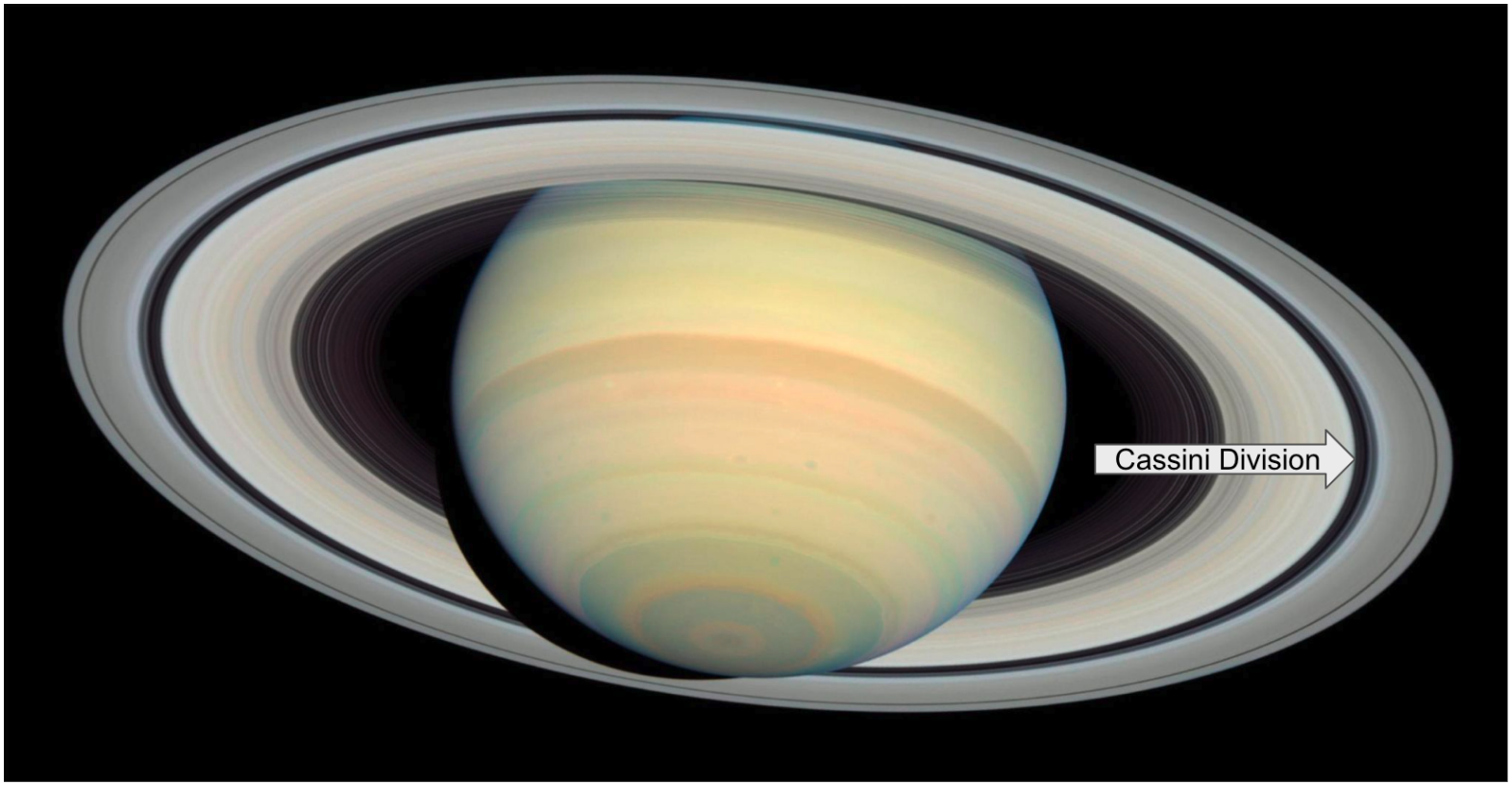
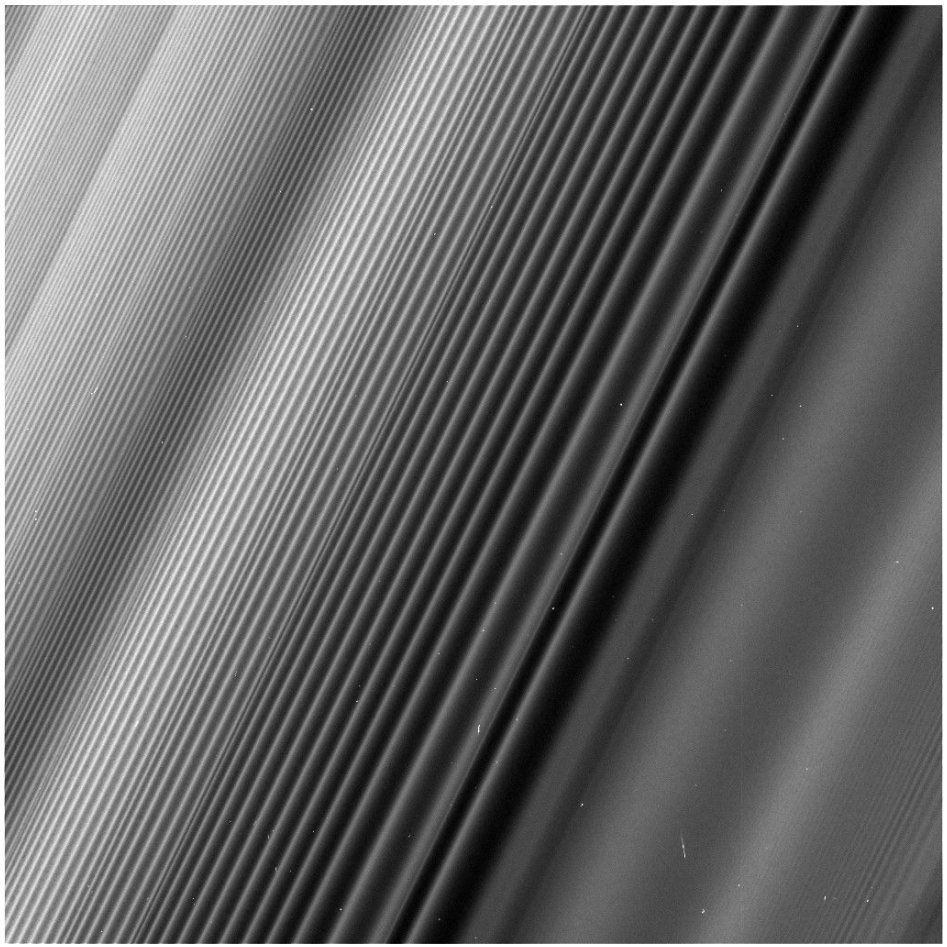
Introduction



Introduction

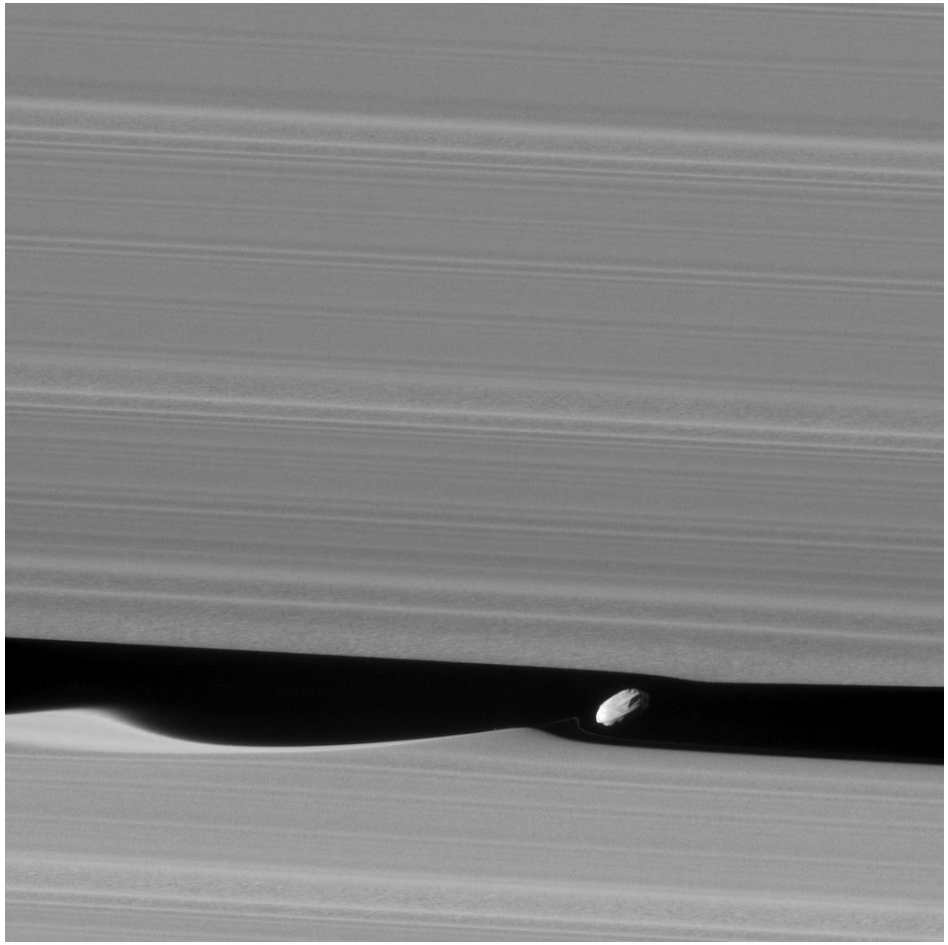


Density Waves



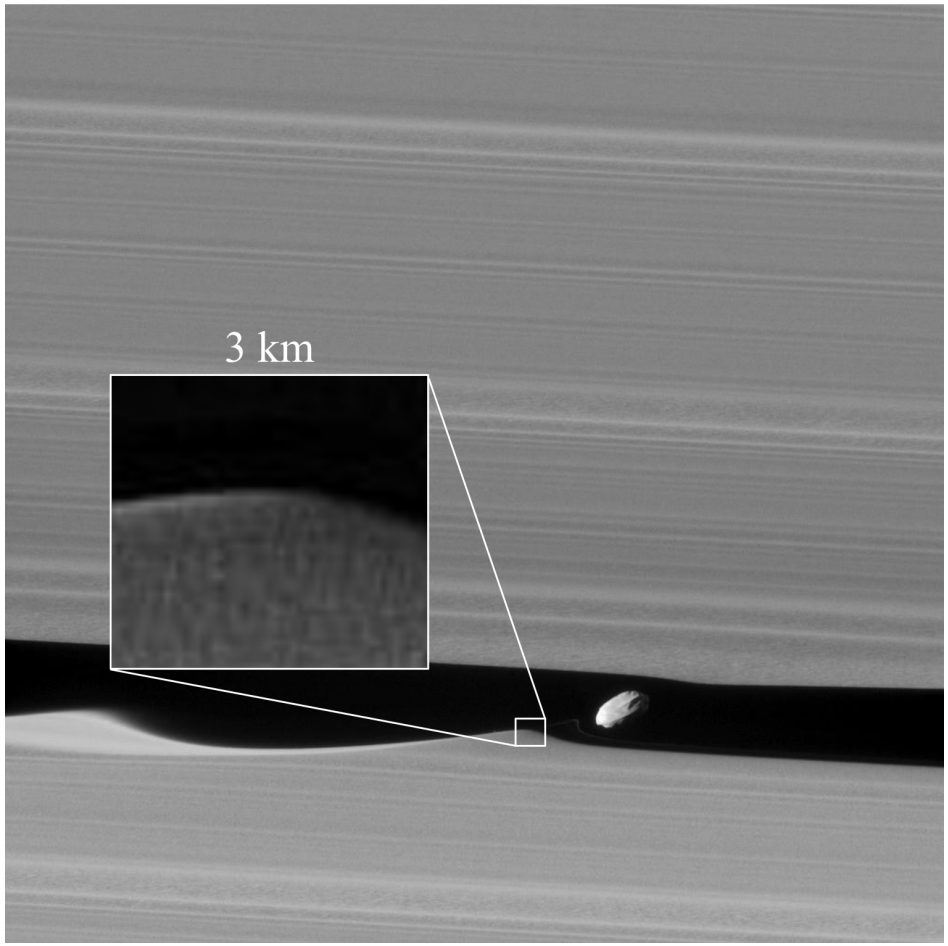
A spiral density wave in the B ring (Janus 2:1)

Vertical Waves



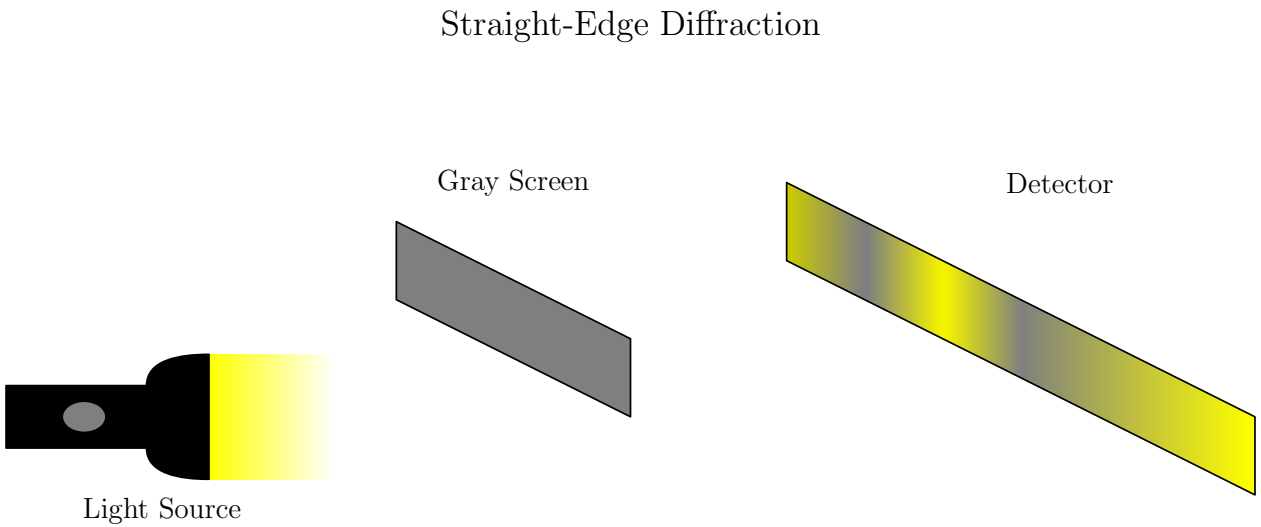
Vertical corrugations left in the wake of Saturn's moon Daphnis

Imaging vs Occultations



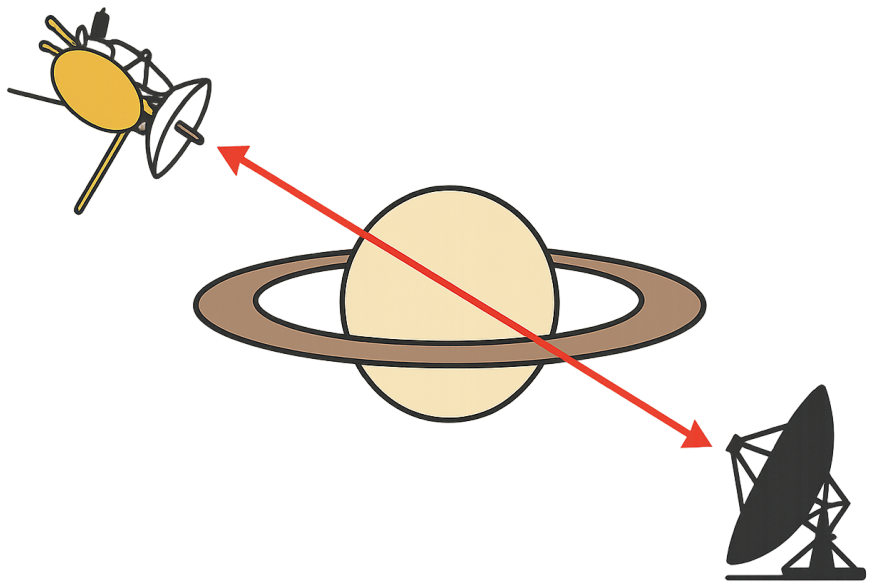
- Radio occultations probe similar and smaller scales in much greater detail than even the sharpest Cassini visual images.

Diffraction



- Waves spread and overlap when passing around obstacles.
- Overlapping waves create bright and dark bands from interference.

Cassini Radio Occultations



- Cassini sends radio waves through the rings to Earth.
- Ring particles attenuate and phase-shift the signal (diffraction).

Scientific Motivation

- Current Cassini radio profiles are limited to 1 km resolution.
- Seek to reconstruct profiles at resolutions of 100 m or better using refined diffraction inversion methods.
- Higher resolutions reveals hidden structure, enables new science.

Ring Diffraction Model

- Crucial relationship between T and $\hat{T}$:

$$\hat{T}(\rho_0) = \frac{\mu_0}{i\lambda} \int_0^{\infty} \rho T(\rho) \int_0^{2\pi} \frac{\exp(i\psi)}{\|\mathbf{R} - \boldsymbol{\rho}\|} d\phi d\rho.$$

- $\mathbf{R}$, ϕ, $\boldsymbol{\rho}$, $\boldsymbol{\rho}_0$, μ_0 detail geometry of observation.
- Applying the stationary phase approximation on the ϕ-integral:

$$\hat{T}(\rho_0) \approx \frac{\mu_0}{i\lambda} \int_0^{\infty} \rho T(\rho) \sqrt{\frac{2\pi}{|\psi_s''|}} \frac{\exp\left(i(\psi_s + \frac{\pi}{4})\right)}{\|\mathbf{R} - \boldsymbol{\rho}_s\|} d\rho.$$

Ring Diffraction Model

- Using the classic Fresnel approximation for ψ_s'':

$$\hat{T}(\rho_0) \approx \frac{1-i}{2F} \int_0^\infty T(\rho) \exp(i\psi_s) d\rho.$$

- The inverse transform is:

$$T(\rho) \approx \frac{1+i}{2F} \int_0^\infty \hat{T}(\rho_s) \exp(-i\psi_s) d\rho_0.$$

Reconstruction Methods

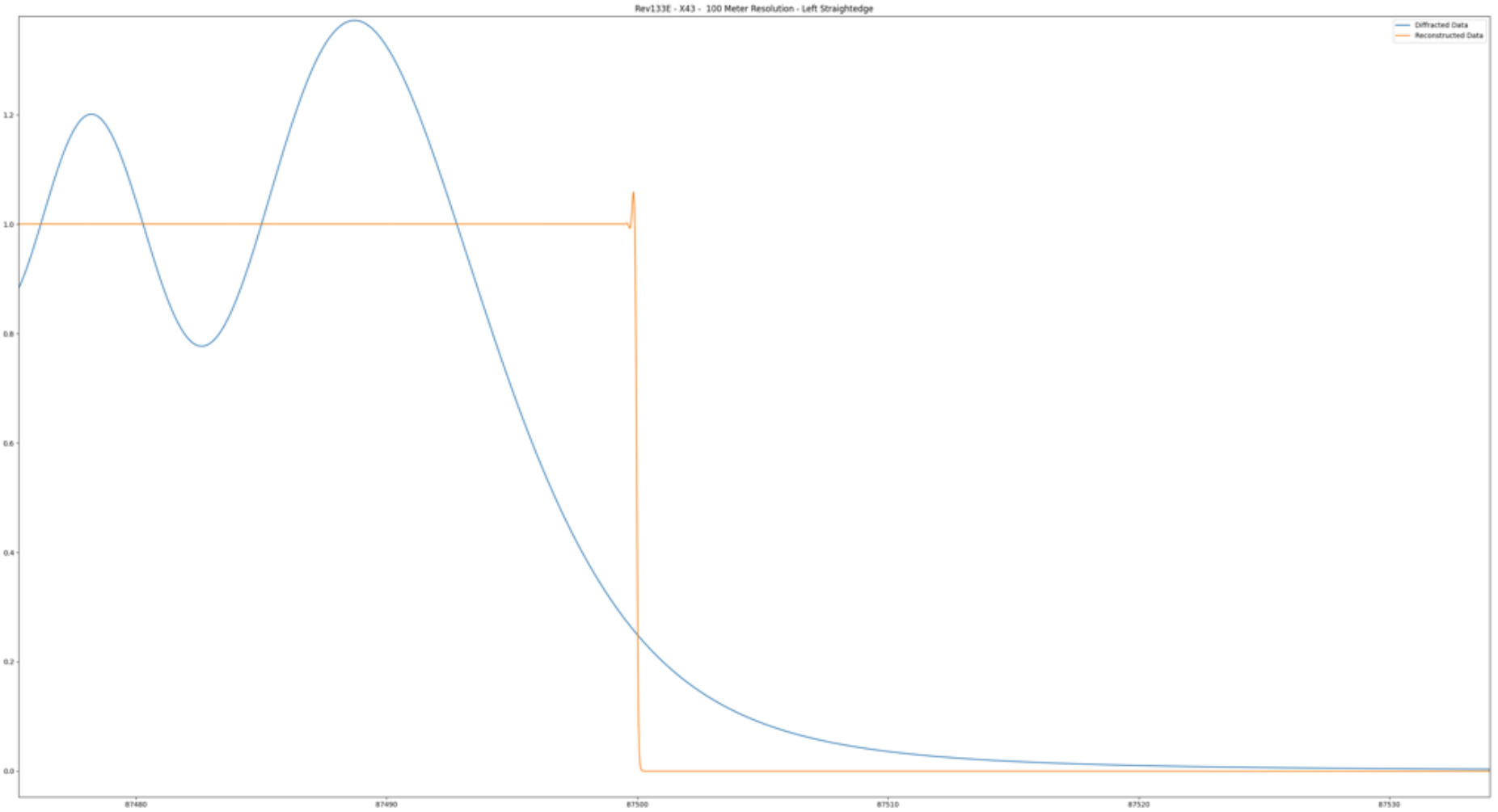
- Inversion is highly dependent on the Fresnel phase ψ_s.
- Fresnel Quadratic (classic): $\psi_s \approx \frac{\pi}{2}\left(\frac{\rho-\rho_0}{F}\right)^2$.
- Octic and Hexadecic Interpolation (new): Approximate ψ_s with eighth and sixteenth order Lagrange interpolated polynomials, respectively.
- Newton (new): Removed many geometric assumptions and approximations.

Reconstruction Methods Overview

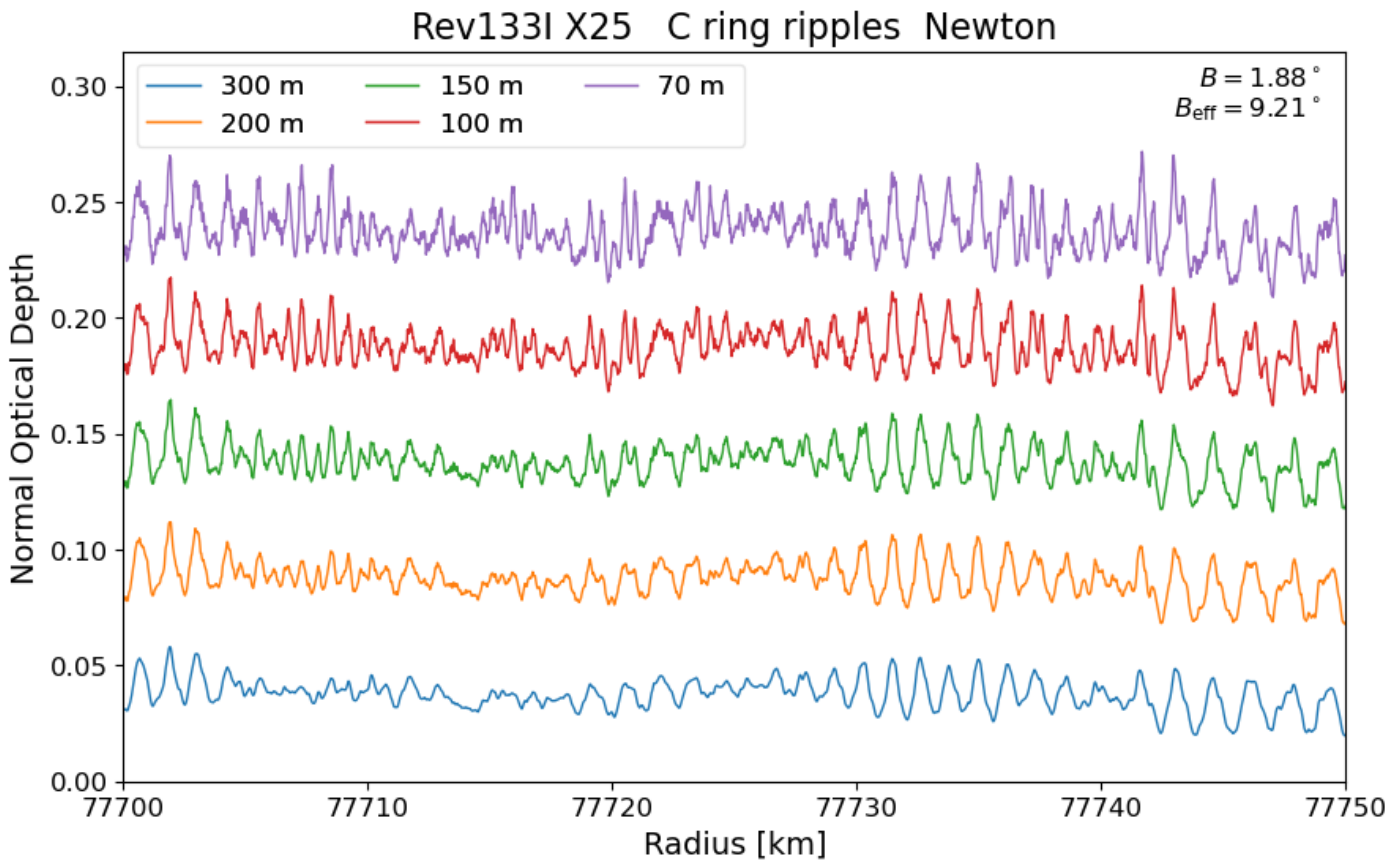
Execution time for processing the Huygens gap at 125 m resolution:

Reconstruction Method	Processing time (seconds)
Fresnel	16
Octic Interpolation	36
Hexadecic Interpolation	41
Newton	419

Forward Model Validation



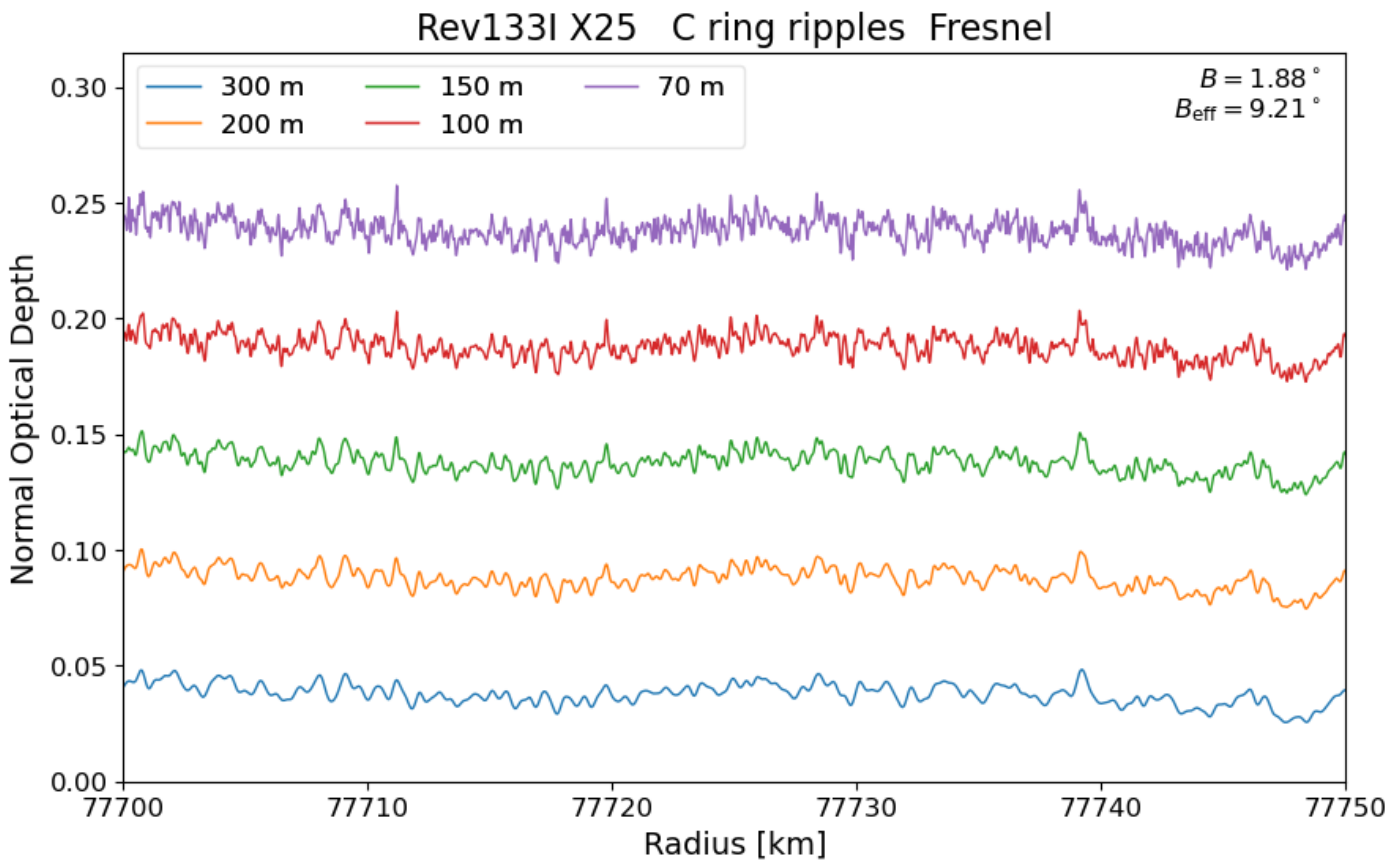
C Ring Ripples



Reconstruction using our new inversion method

- Low-amplitude beating pattern from interference of two oscillating frequencies

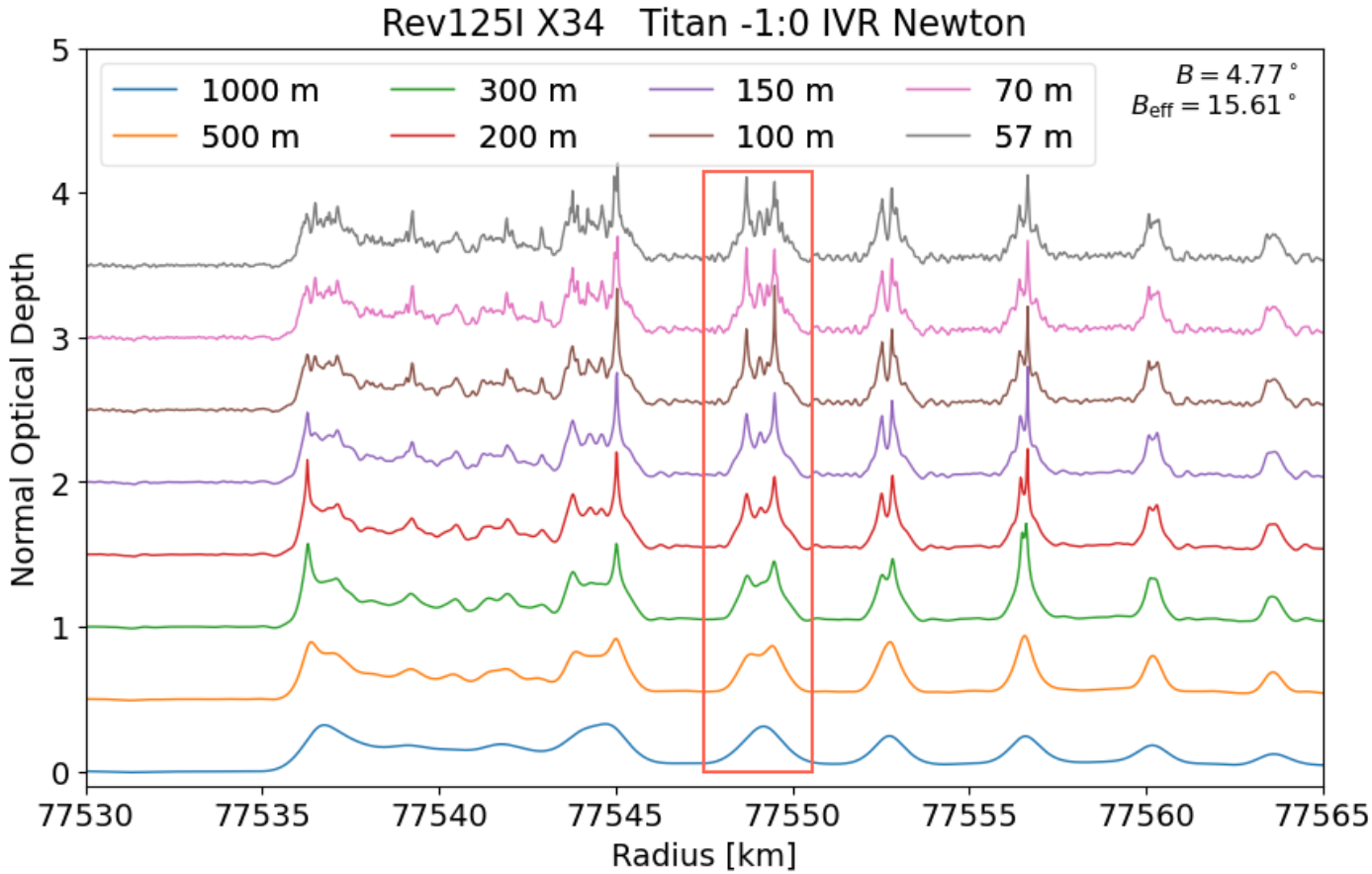
C Ring Ripples (Fresnel)



Reconstruction using Fresnel Quadratic approximation of ψ

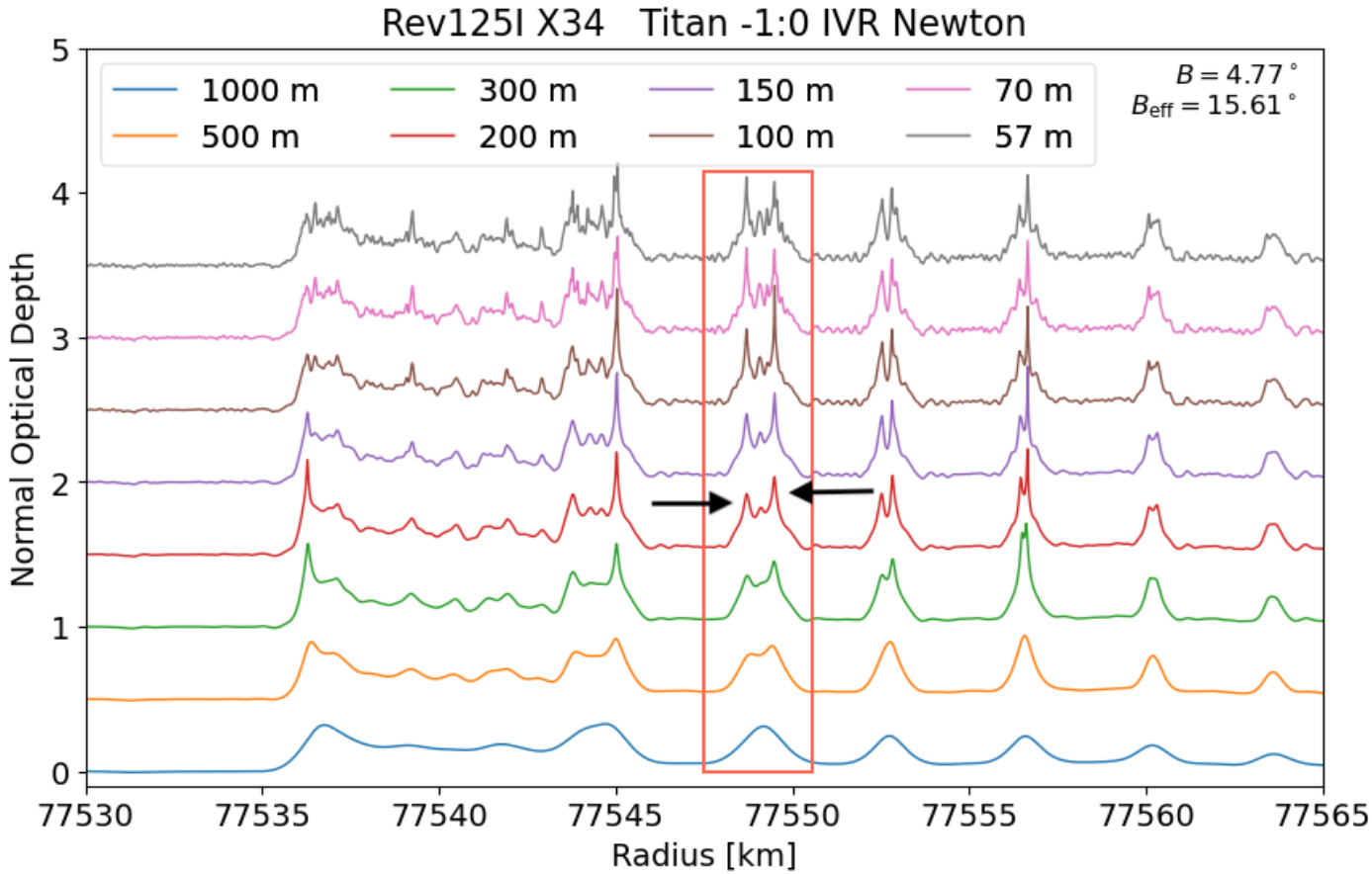
- Dominated by noise, zero visible structure

Titan Bending Wave



- Multiple cusps within a single wave crest indicate multiple vertical crossings by the occultation ray.

Titan Bending Wave



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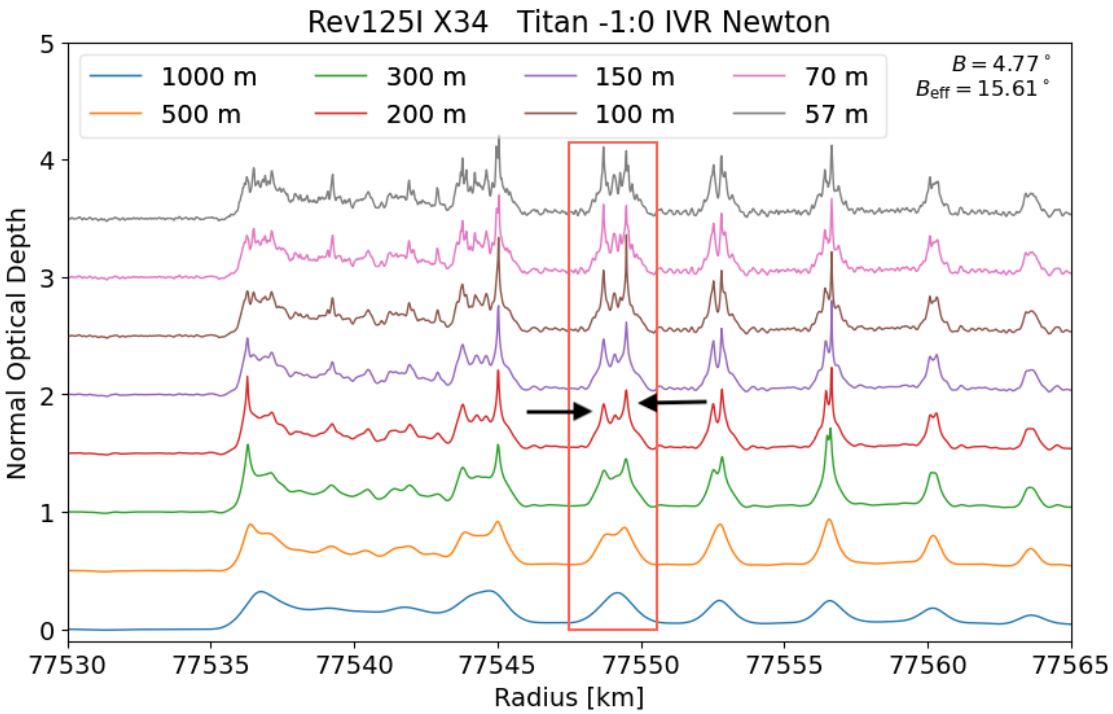
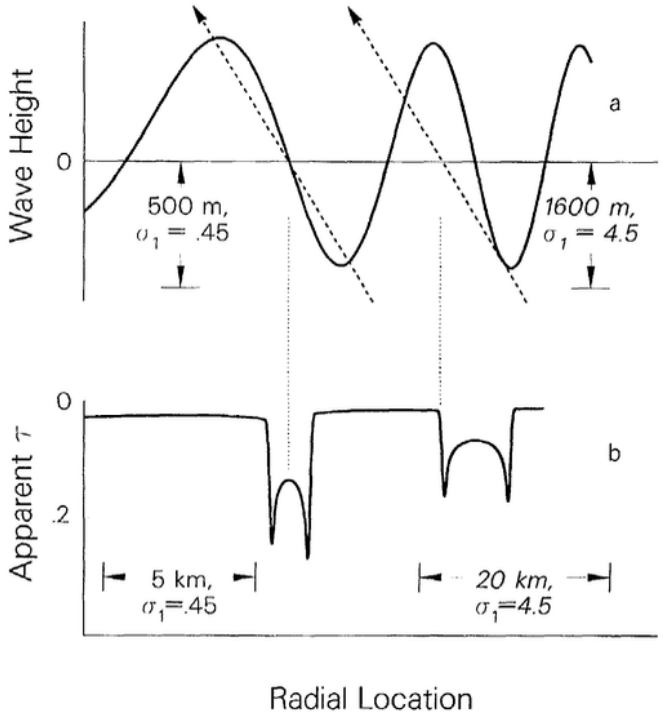
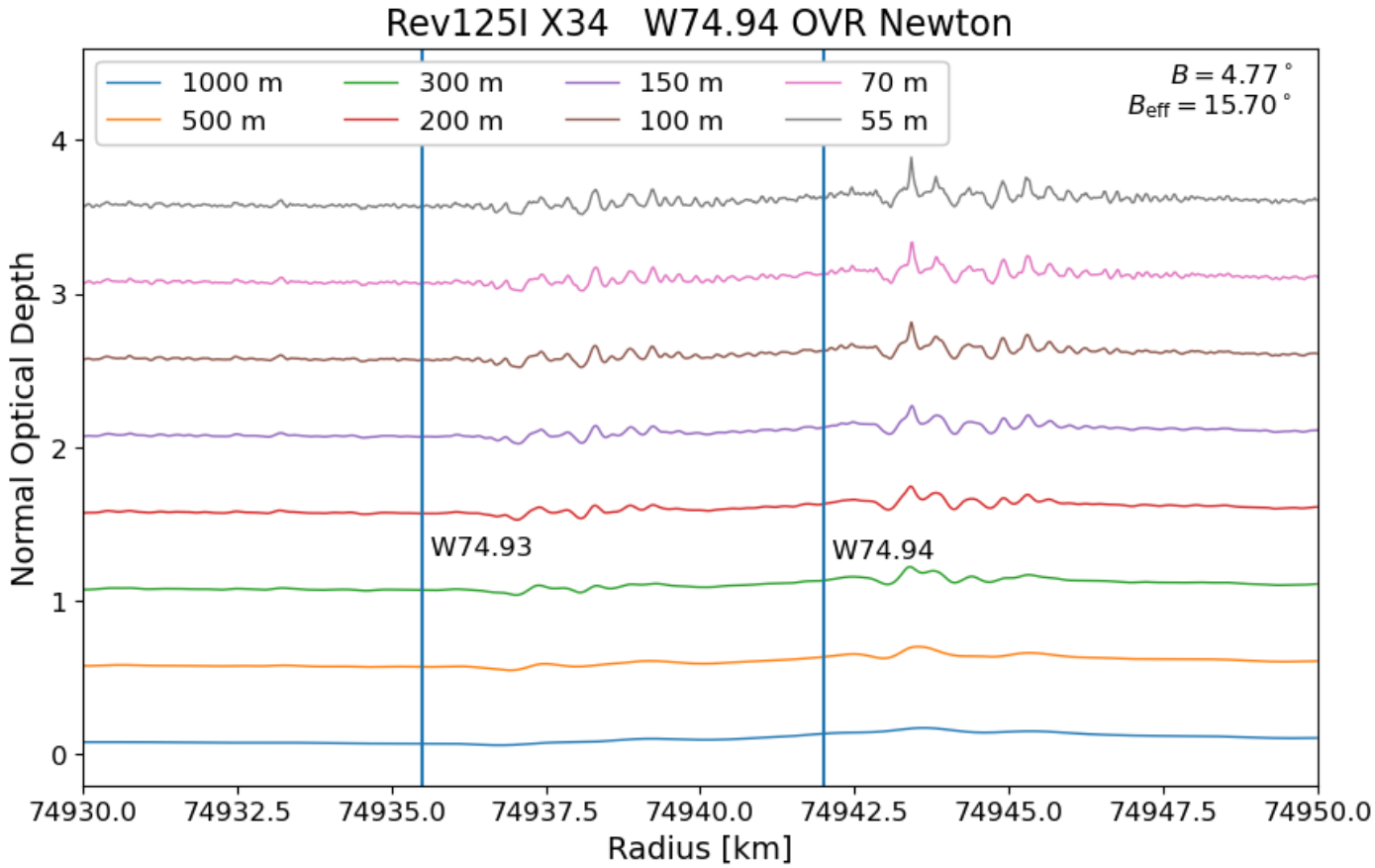




Optical depth increases downwards in the diagram at right.

- Multiple cusps within a single wave crest indicate multiple vertical crossings by the occultation ray.

A Pair of Bending Waves



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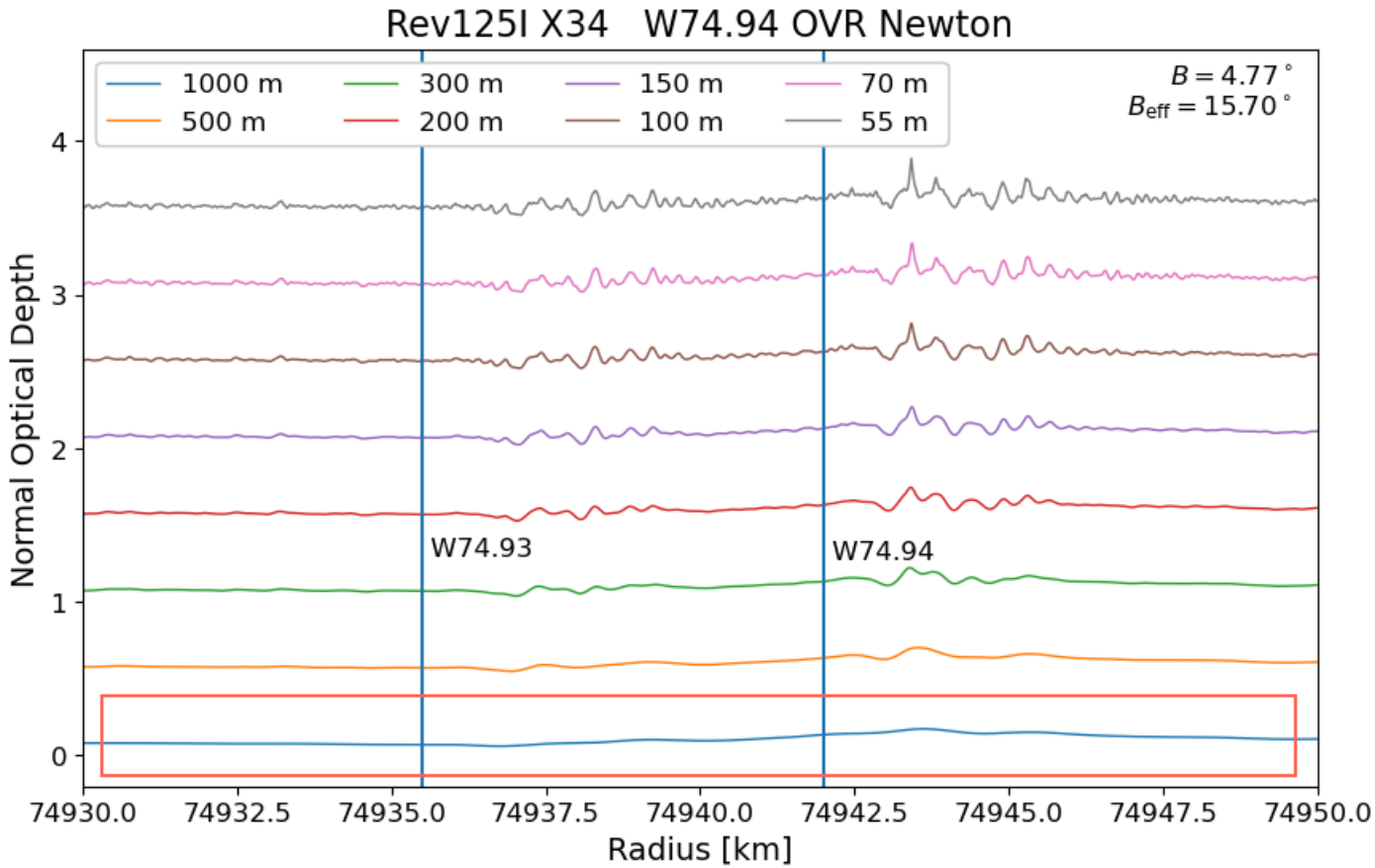
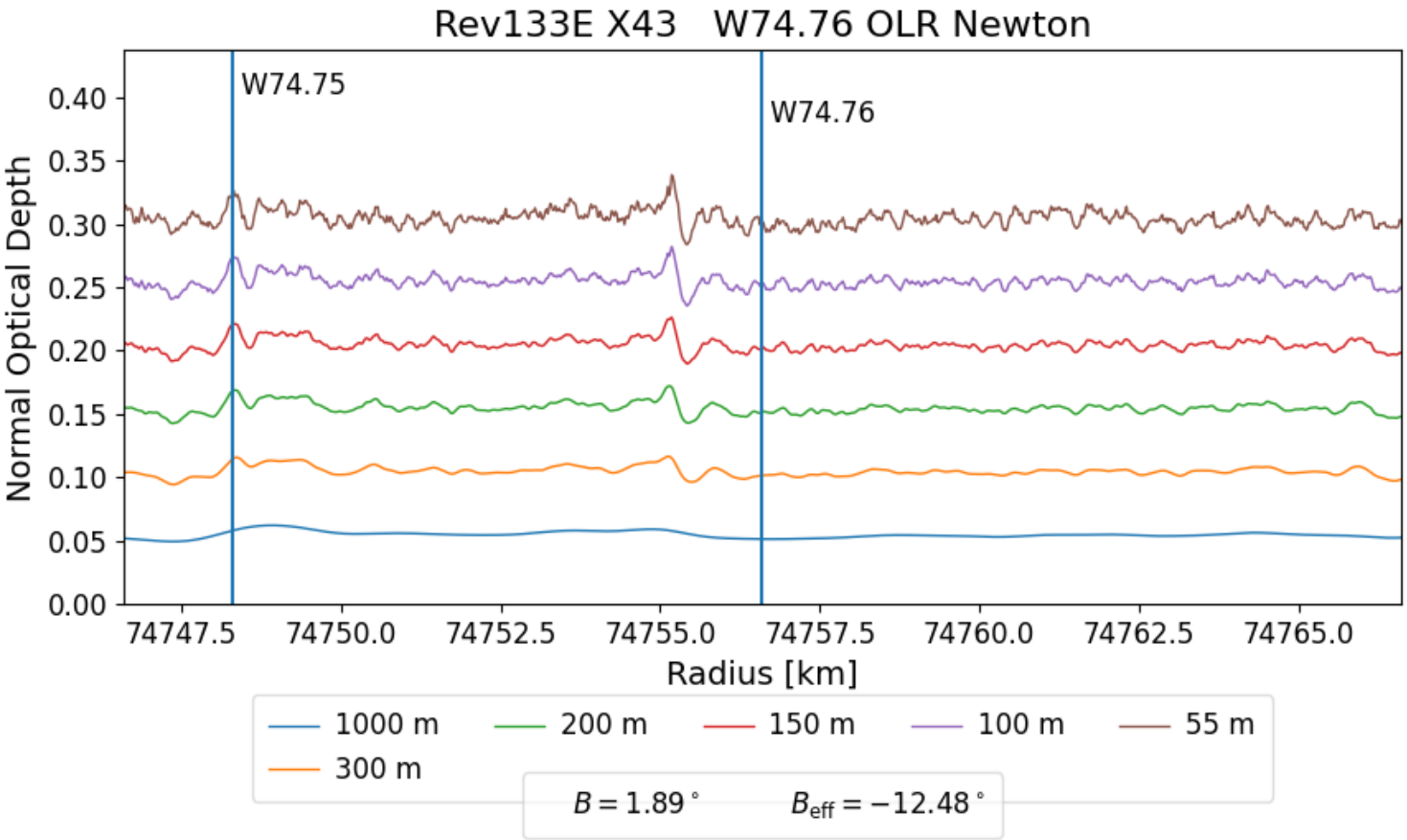
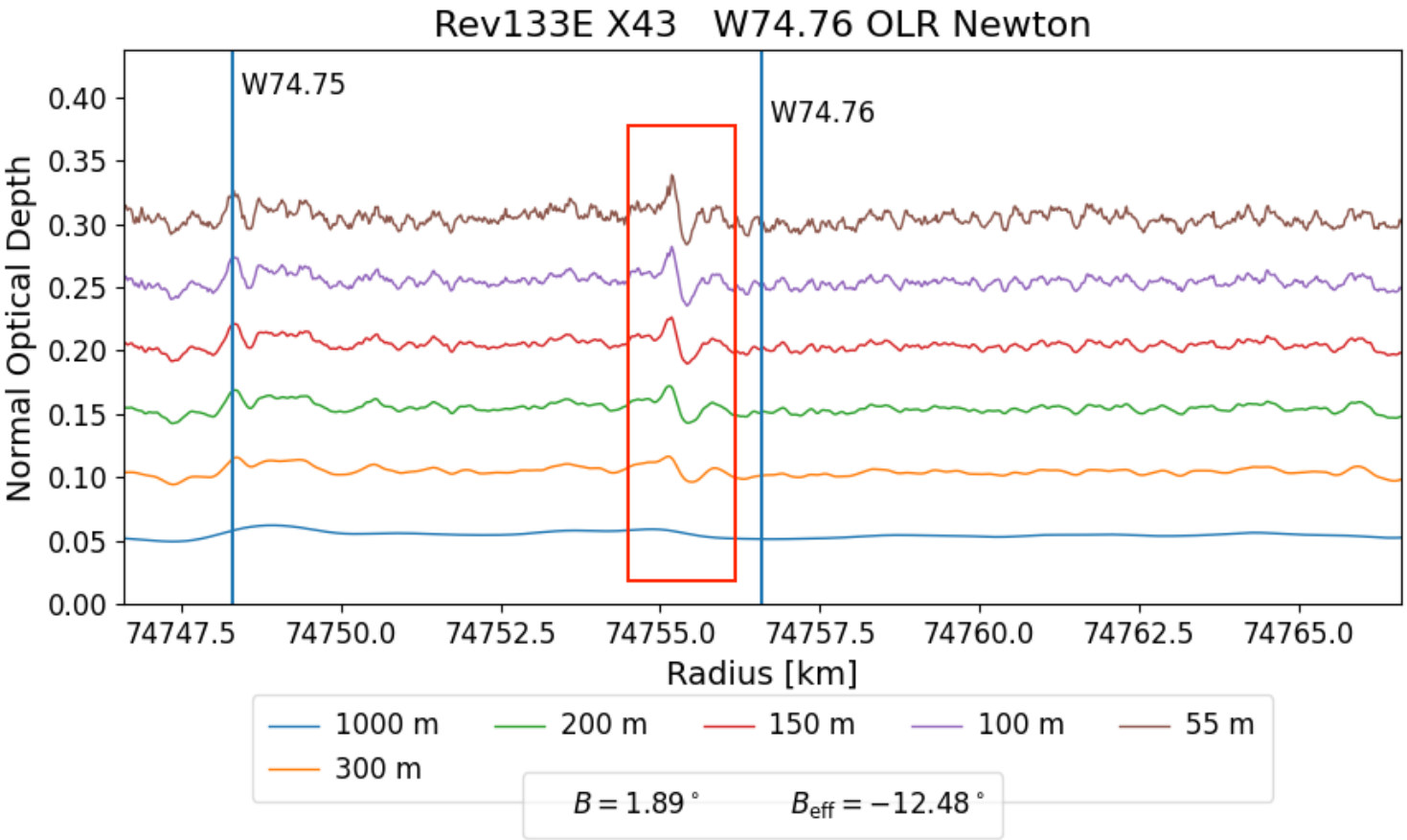


- **Invisible** in existing NASA archived products
- Low noise even at 55 m resolution

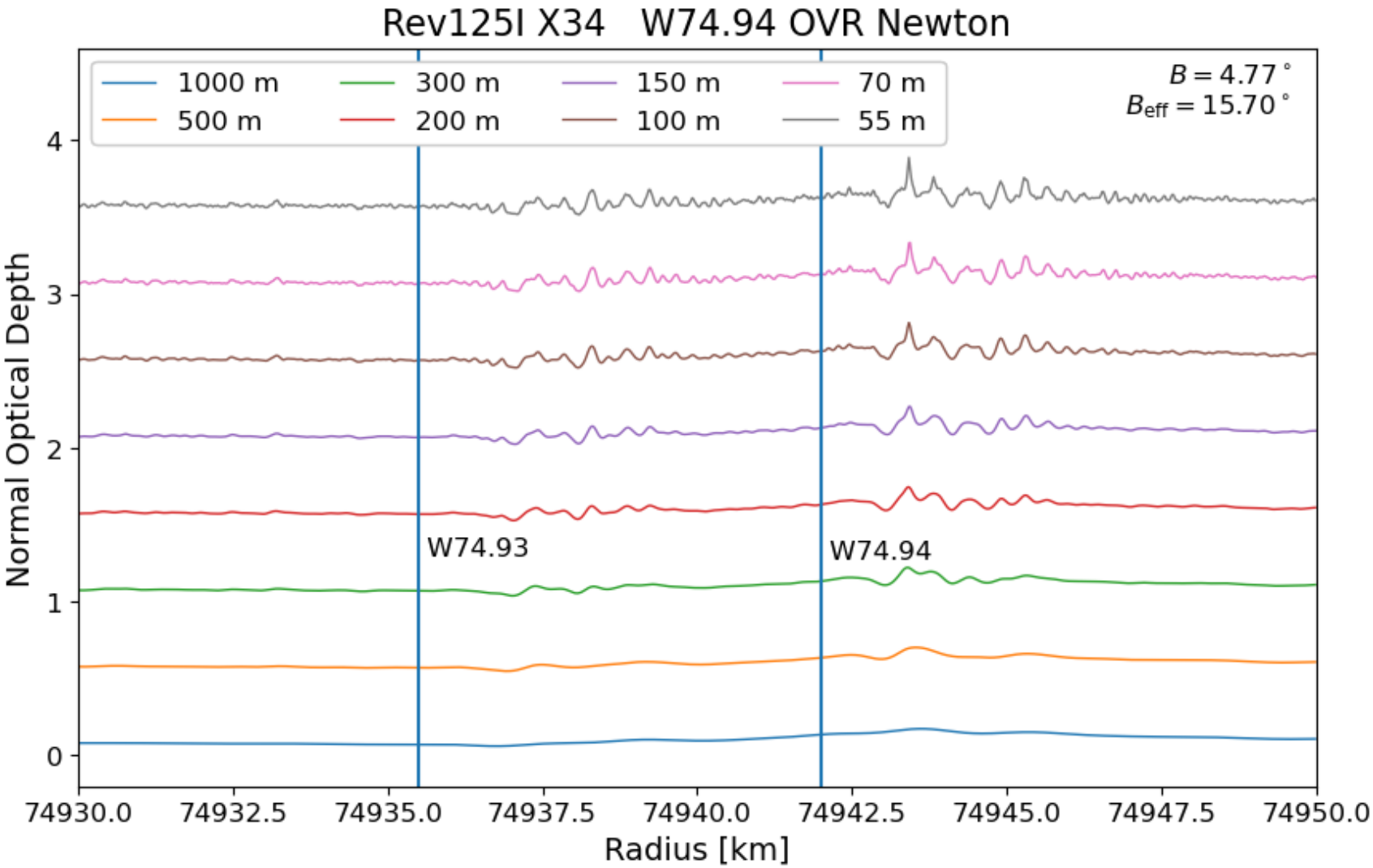
Density Wave Near Inner C Ring



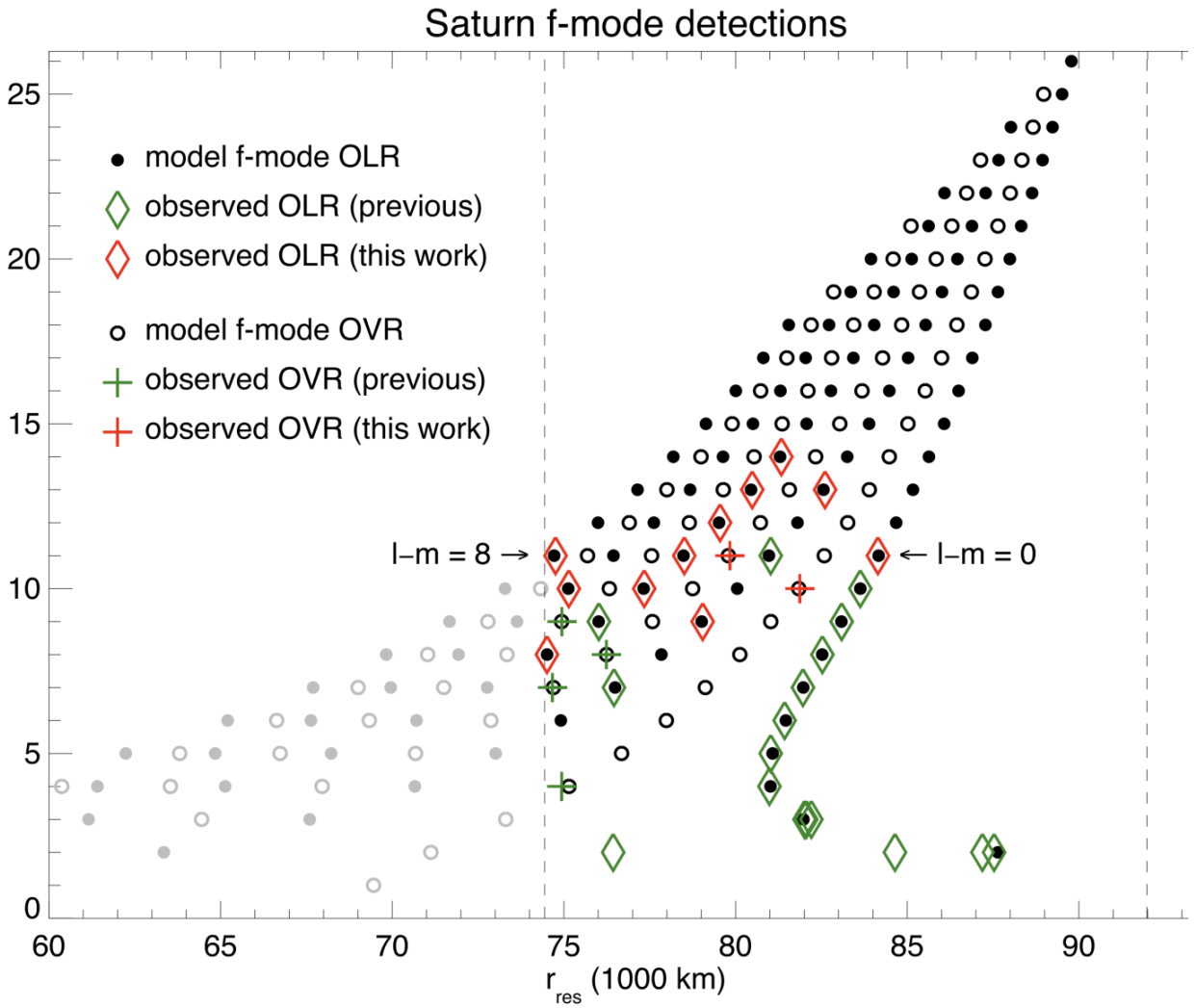
Density Wave Near Inner C Ring



Conclusion


Rev125I X34 W74.94 OVR Newton
1000 m
500 m
300 m
200 m
150 m
100 m
70 m
55 m
$B = 4.77^\circ$
$B_{\text{eff}} = 15.70^\circ$
W74.93
W74.94
Normal Optical Depth
0
1
2
3
4
74930.0
74932.5
74935.0
74937.5
74940.0
74942.5
74945.0
74947.5
74950.0
Radius [km]

Conclusion



Predicted and observed wave resonance locations in Saturn's C Ring

Key Takeaways

- We beat diffraction in practice: ≤ 100 m reconstructions, down to 55 m in cases.
- Resolves fine ring features invisible in prior Cassini profiles
- Better census of faint seismology waves leads to constraints on Saturn's interior.

Acknowledgements

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- Dr. Richard French
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References I

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[3] NASA/JPL-Caltech/Space Science Institute. *PIA21627: Staggering Structure*. 2017. URL: https://photojournal.jpl.nasa.gov/catalog/PIA21627.

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[5] NASA/JPL-Caltech/Space Science Institute. *PIA05982: Saturn from Far and Near*. 2004. URL: https://photojournal.jpl.nasa.gov/catalog/PIA05982.

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