Sparse Inference of Nonlinear Laboratory Earthquake Dynamics

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Motivation

2011 Tohoku earthquake and tsunami

Figure: Sagiya et al. [6]

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2011 Tohoku earthquake and tsunami

- Magnitude 9.0; 19,759 confirmed dead [2]

Figure: Sagiya et al. [6]

What are "Labquakes?"

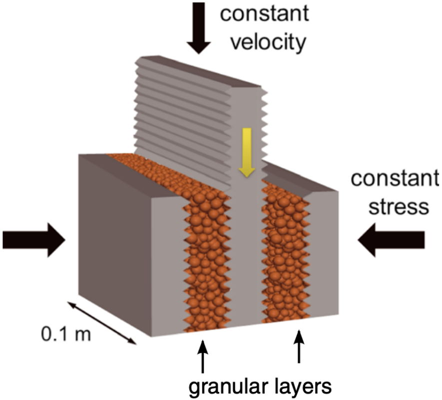

Figure: Johnson et al. [4]

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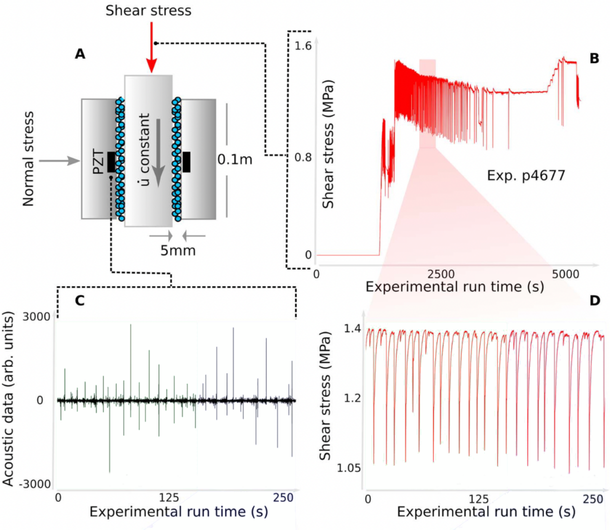

Figure: Rouet-Leduc et al. [5]

Goals

We aim to analyze the frictional states of laboratory earthquakes ("labquakes") through acoustic emission (AE) data, guided by physical laws.

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- Real earthquakes result from friction along geological faults
- Understand the nuances of frictional state evolution for labquakes to shed light on real events

Analyzing Dynamical Systems

We use ordinary differential equations (ODEs), partial differential equations (PDEs), or stochastic differential equations (SDEs) to model data.

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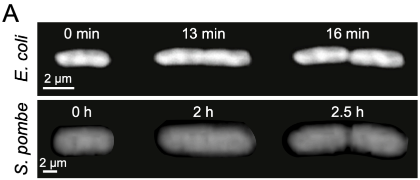

Figure: Zhang et al. [7]

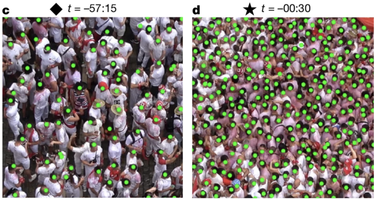

Figure: Gu et al. [3]

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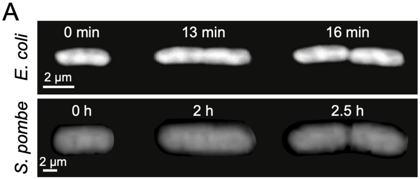

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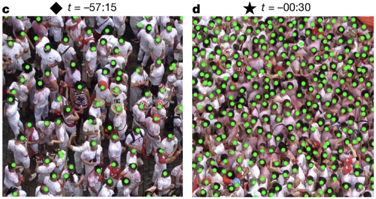

Figure: Gu et al. [3]

$$ds_t = g(s_t)dt - h(s_{t-})d\mathcal{N}_t(\lambda_t)$$

$$\partial_t \mathbf{u} = -\frac{k}{\gamma}\mathbf{u} + \frac{1}{\gamma}\mathbf{p} + \frac{\sigma}{\gamma}\zeta,$$

$$\partial_t \mathbf{p} = -\gamma_{\mathbf{p}}\mathbf{p} + \beta\gamma_{\mathbf{p}}\left(1 - \frac{\eta}{\gamma_{\mathbf{p}}}\mathbf{p}^2\right)\partial_t \mathbf{u}$$

$$-\alpha^2(\mathbf{p} \times \partial_t \mathbf{u}) \times \mathbf{p} + \sigma_{\mathbf{p}} + \zeta_{\mathbf{p}}.$$

Nonlinear Inference Methods

Sparse Identification of Nonlinear Dynamics (SINDy)

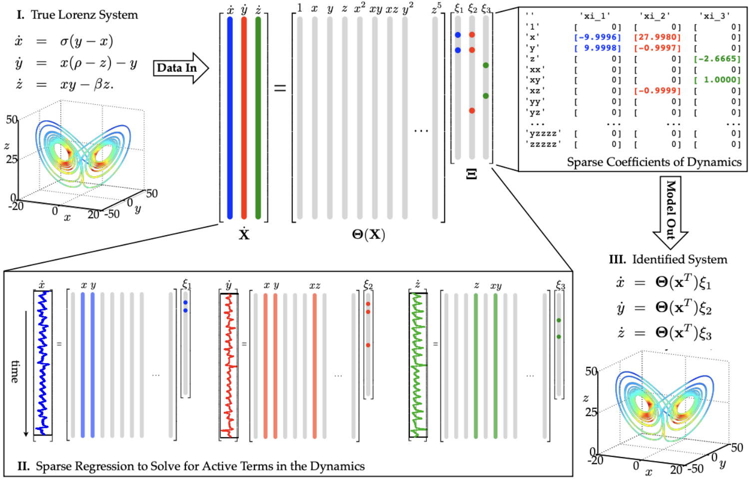

Figure: Brunton et al. [1]

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Frictional States

Definition

A labquake's *frictional state* θ is a governing state variable that grows over time, dictating the time-evolution behavior of the frictional coefficient μ.

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Rate-and-State Friction Equation (Dieterich-Ruina)

In a labquake, μ is as follows:

$$\mu = \mu_0 + a \ln\left(\frac{V}{V_0}\right) + b \ln\left(\frac{V_0 \theta}{D_c}\right),$$

where μ_0 is the reference friction coefficient, V is the slip velocity, V_0 is a reference velocity, D_c is the characteristic slip distance over which friction evolves, and a and b are empirically-determined constants.

Friction Laws

Perrin, Rice, and Zheng (PRZ) Friction Law

$$\frac{d\theta}{dt} = 1 - \left(\frac{V\theta}{2D_c}\right)^2.$$

PRZ takes into account both aging and slip effects.

Change in μ from PRZ

$$\frac{d\mu}{dt} = -\frac{bV}{D_c}\sinh\left(\frac{\mu-\mu_0}{b} + \ln\left(\frac{1}{2}\cdot\left(\frac{V}{V_0}\right)^{1-\frac{a}{b}}\right)\right)$$

Modeling $\frac{d\mu}{dt}$ with Hyperbolic Sine

Change in μ from PRZ

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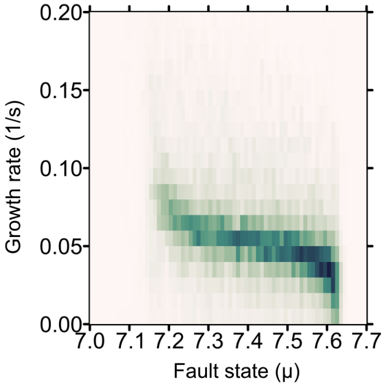

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From Data to Governing Laws

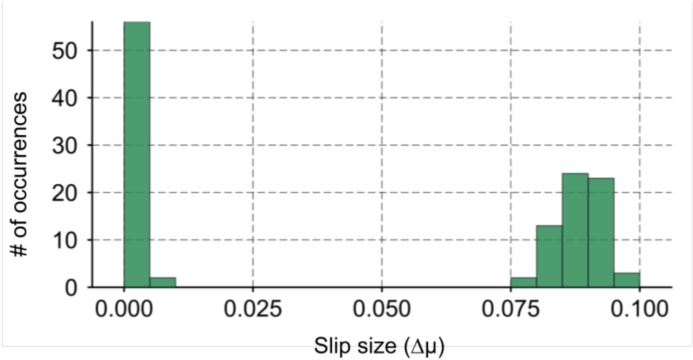
- Goal: infer governing stochastic dynamics directly from time-series.
- Many natural systems show gradual growth + sudden releases.
- Model class: SDE with Poisson jump events to capture sudden drops.

Stochastic jump SDE

$$ds_t = g(s_t)\,dt - h(s_{t-})\,d\mathcal{N}_t(\lambda_t)$$

- $g(s_t)$: deterministic drift (slow build-up).
- $h(s_{t-})$: jump size at events.
- $\mathcal{N}_t(\lambda_t)$: inhomogeneous Poisson process with rate λ_t.
  - Poisson events encode random event timing (with history dependence).

Microslips and Major Slips



SDE Inference for Labquakes

SDE Model for Labquakes

We may model the change in the frictional coefficient μ with an SDE as follows:

$$d\mu_t = g(\mu_t)dt - h^m(\mu_{t-})d\mathcal{N}_t(\lambda_t^m) - h^M(\mu_{t-})d\mathcal{N}_t(\lambda_t^M),$$

where $h^m(\mu_{t-})$ and λ_t^m are the jump size and rate function for microslips, and $h^M(\mu_{t-})$ and λ_t^M are the jump size and rate function for major slips.

- We calculate $h^m(\mu_{t-})$ and $h^M(\mu_{t-})$ from the empirical distribution of stress drops.

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- We calculate $h^m(\mu_{t-})$ and $h^M(\mu_{t-})$ from the empirical distribution of stress drops.
- We calculate $g(\mu_t)$ with the ADAM-SINDy approach.

ADAM-SINDy Inference

Change in μ from PRZ

$$g(\mu) = \frac{d\mu}{dt} = -\frac{bV}{D_c} \sinh\left(\frac{\mu - \mu_0}{b} + \ln\left(\frac{1}{2} \cdot \left(\frac{V}{V_0}\right)^{1-\frac{a}{b}}\right)\right)$$

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Given the data and the equation $\frac{d\mu}{dt} = \alpha \sinh(\beta\mu + \gamma)$, we want to find α, β, and γ. Then we can derive constants.

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- Problems with standard SINDy

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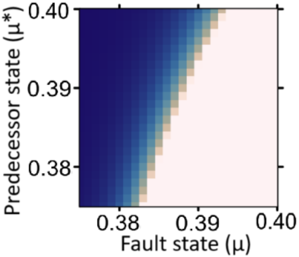
Given the data and the equation $\frac{d\mu}{dt} = \alpha \sinh(\beta\mu + \gamma)$, we want to find α, β, and γ. Then we can derive constants.

- Problems with standard SINDy
- ADAM-SINDy: combine SINDy with Adam optimization

History Dependence

State/history-dependent rate

$$\lambda_t = \lambda(s_t, s_t^*)$$



$\lambda(s_t, s_t^*)$ at $\sigma_n = 10$MPa

- s_t^*: "memory" state just before the last event (one-generation).
- Captures how current and pre-event conditions shape event likelihood.

Parameterizing the Event Rate

- Enforce $\lambda > 0$ by modeling its log-rate.

Orthogonal basis expansion

$$\ln \lambda(s_t, s_t^*) = \sum_{i,j} w_{ij}\, \theta_i(s_t)\, \theta_j(s_t^*)$$

- $\{\theta_i\}$: orthogonal polynomials (via modified Gram–Schmidt) on observed states.
- w_{ij}: coefficients to learn; we will promote sparsity.

Sparse Bayesian Inference

Bayes' Theorem

$$P(\mathbf{w} \mid \text{data}) = \frac{P(\text{data} \mid \mathbf{w})\, P(\mathbf{w})}{P(\text{data})}$$

- Posterior $\propto$ Likelihood $\times$ Prior.
- Normalizing constant $P(\text{data})$ can be ignored in inference.

Sparsity-promoting Gaussian prior

$$w_{ij} \sim \mathcal{N}(0,\ \sigma_{ij}) \quad \Rightarrow \quad P(\mathbf{w}) \propto \prod_{i,j} \exp\left(-\frac{w_{ij}^2}{2\sigma_{ij}^2}\right)$$

- Expectation–Maximization adjusts σ_{ij} iteratively.
- Shrinks irrelevant w_{ij} toward 0, promoting sparsity.

Likelihood for Inhomogeneous Poisson Jumps

- Observations: a continuous trajectory with event times $\{\tau_k\}$.

Log-likelihood

$$\ln P(\text{data} \mid \mathbf{w}) = -\int \lambda(s_t, s_t^*)\, dt + \sum_k \ln \lambda\left(s_{\tau_k}, s_{\tau_k}^*\right)$$

- First term: probability of no events between jumps.
- Second term: contribution from each observed jump time.

Maximum a posteriori estimation

$$\hat{\mathbf{w}} = \arg\max_{\mathbf{w}} \left\{ P(\mathbf{w} \mid \text{data}) \right\}$$

- Find most probable coefficients $\hat{\mathbf{w}} = \{w_{ij}\}$ given observed data.

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Datasets and Sampling

- Low σ_n (4–8 MPa): experiment p2394
- Low σ_n have microslips and major slips

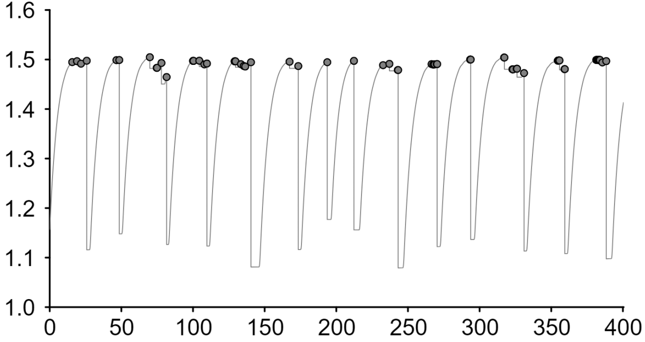


Figure: Shear stress trajectory for 4 MPa

Datasets and Sampling

- High σ_n (9–13 MPa): experiments p4346, p4347, p4348, p4350
- High σ_n only have major slips.

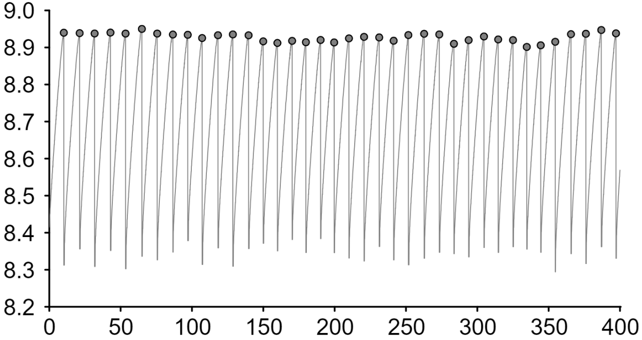


Figure: Shear stress trajectory for 13 MPa

Denoising and Event Detection

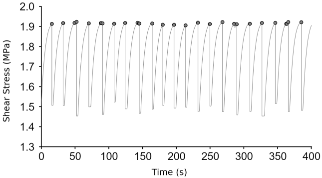
- Apply total variation (TV) denoising to preserve sharp drops.

Total variation (TV) denoising

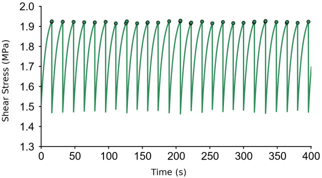
$$\min_{x} \left(\|x - y\|_2^2 + \lambda_{\mathrm{TV}} \sum_{t} |x_t - x_{t-1}| \right).$$

- 4–8 MPa: detect slips via local maxima $\rightarrow$ next local minima to capture microslips.
- 9–13 MPa: remove intra-drop points; threshold amplitude to only keep major slips.

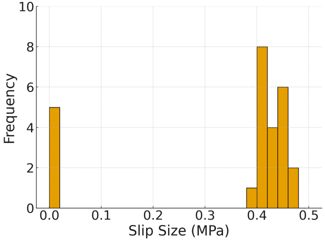
Results at 5 MPa



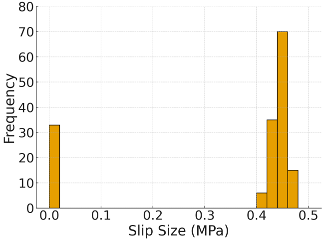
(a) Original 5 MPa



(b) Simulated 5 MPa

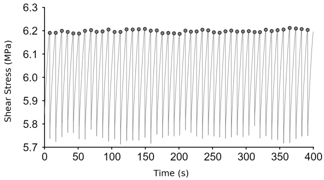


(c) Original 5 MPa

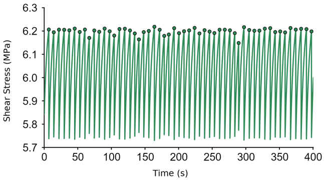


(d) Simulated 5 MPa

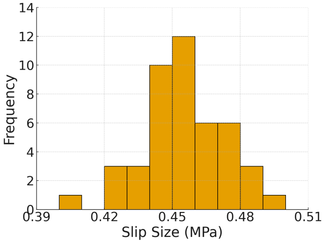
Results at 9 MPa



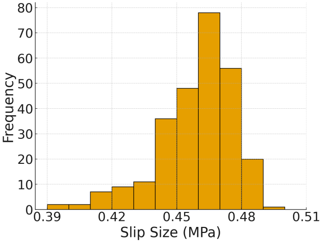
(a) Original 9 MPa



(b) Simulated 9 MPa

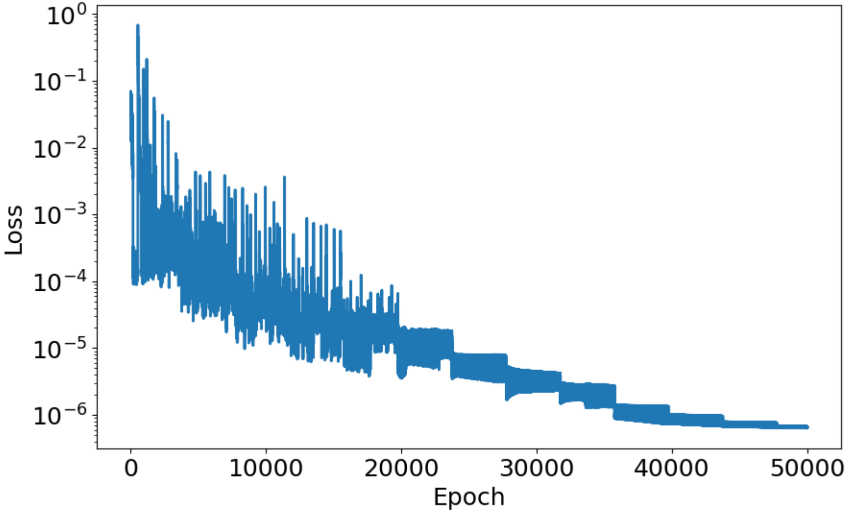


(c) Original 9 MPa



(d) Simulated 9 MPa

Growth Function Inference



Conclusion

- SDE modeling allows for more insights than machine learning
- Dual-process model for labquakes (microslips and major slips)
- Synthesis of experimental data and theory through sinh growth function
- Inference using ADAM-SINDy

Acknowledgments

We would like to thank the following individuals and organizations:

- Our mentor, Shijie Zhang, for suggesting the project and guiding us along the way;
- The MIT PRIMES-USA program for their continual support on this research;
- Our former collaborator, Jonathan Du, for providing insights on ridge regression techniques;
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- Dr. Tanya Khovanova and Alyssa Yu for giving us invaluable feedback on this presentation;
- Our family members for supporting us!

Model Parsimony and Selection

- Sequential thresholding: prune small w_{ij}, refit.
- Evaluate a family of sparse candidate models.

Bayesian Information Criterion (BIC) score

$$\mathrm{BIC} = \ln P(\text{data} \mid \hat{\mathbf{w}}) - \frac{1}{2} \ln \left| \hat{\mathbf{H}} \right|$$

- $\hat{\mathbf{H}}$: Hessian of $\ln P(\hat{\mathbf{w}} \mid \text{data})$
- Penalizes candidate models with lots of parameters.
- Select the model with the largest BIC.

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