Surfaces in Knot Theory

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MIT PRIMES Conference 2025

May 18, 2025

What is Knot Theory?

Knot theory is a subfield of topology.

A knot is a closed, non-intersecting curve embedded in 3-dimensional space.

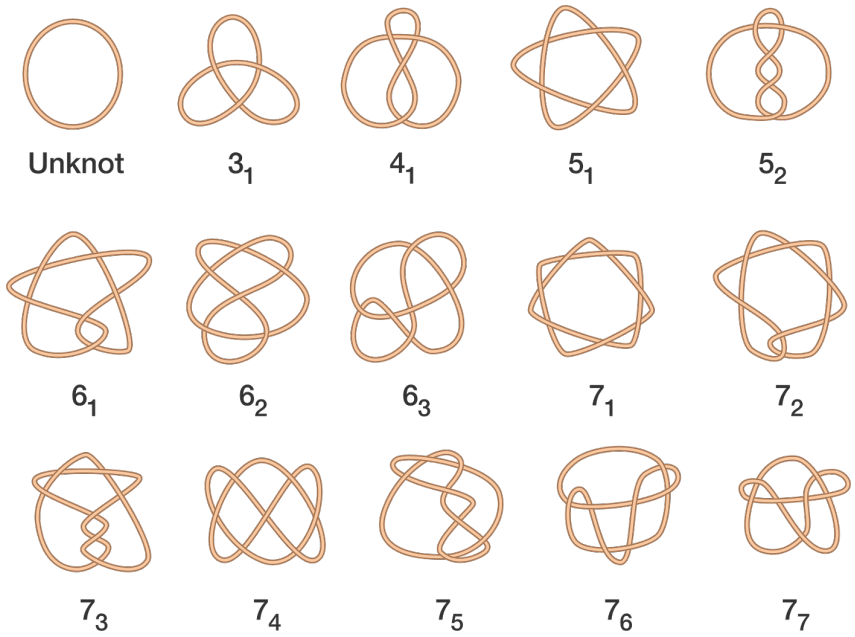

Figure: Knots

Central Question in Knot Theory

How do we distinguish knots? This question often arises because knots can be deformed into each other through continuous transformations, known as ambient isotopy.

Knot invariants are quantities that remain unchanged under ambient isotopy. These quantities also serve as tools to classify and distinguish different knots.

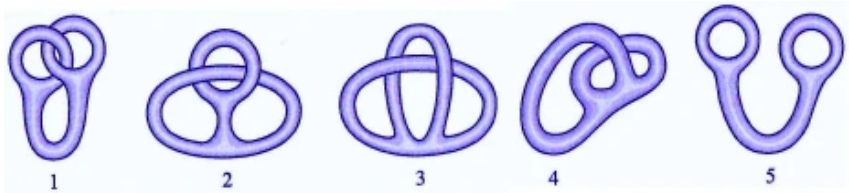

Figure: Example of Ambient Isotopy

Surfaces - What is a Surface in Knot Theory?

A surface is a two-manifold.

Manifold: An n-dimensional manifold is an object for which the neighborhood of any point on the manifold looks like n-dimensional space.

Some examples of surfaces include:

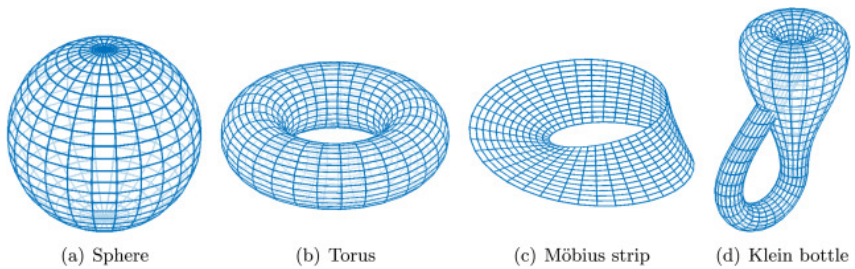

Figure: Surfaces

Seifert Surfaces

Seifert Surfaces are orientable surfaces that have one boundary component so that the boundary is the knot. Orientable surfaces are surfaces upon which you can color one side black, the other side white, and neither side will touch except along the border. You can make Seifert Surfaces by taking an alternating projection of an alternating knot and using Seifert's equation on it.