The Basics of Computation

Shamini Biju, ZZ Zhang, and Sherri Wu

MIT Primes Circle

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Overview

1. Introduction
2. Languages
3. Finite Automata
4. Deterministic Finite Automata
5. Nondeterministic Finite Automata
6. Different, But Equivalent: DFA vs NFA
7. Real World Applications

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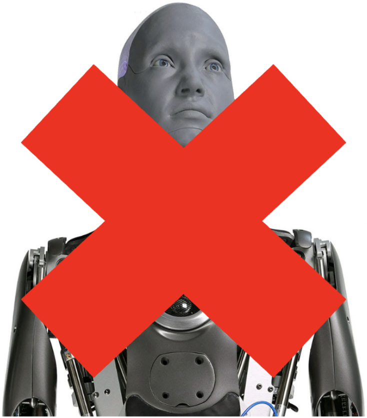
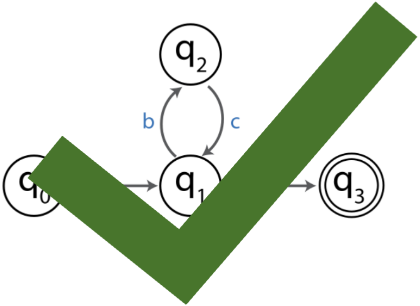
Introduction

- Computation theory uses different **machines** to explore what computers can and cannot do by recognizing different classes of languages

Machines





Introduction

Definition of a Machine

A machine is a mathematical model that processes inputs based on a set of rules to produce outputs. It follows specific instructions, step-by-step, to determine whether a given input is accepted or rejected.

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- Computation theory uses different **machines** to explore what computers can and cannot do by recognizing different classes of languages
- Finite Automata are the simplest machines, recognizing basic patterns in strings
- More advanced machines like CFGs, PDAs, and Turing Machines handle complex structures and algorithms

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Regular Language Example:
String over the alphabet (0,1) that end in 01

What types of strings might be in this language?

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- Think of a language like a recipe book.
  - Each recipe is a string of instructions made up of a specific set of ingredients (symbols).
  - A language defines which strings are "valid" inputs for a computational problem.
  - It only "accepts" recipes (strings) that follow the defined format.

Language Venn Diagram

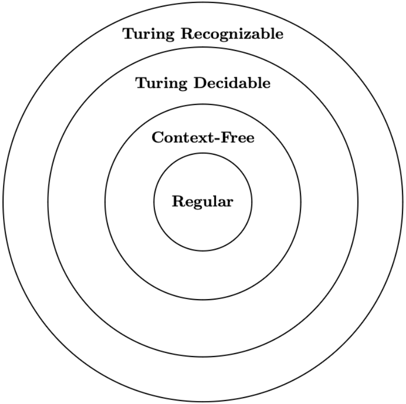

Figure: We will be talking about regular languages today.

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- Recognizing a language: a machine takes an input and determines whether the string belongs to the language
- Finite Automata are the simplest model out of more advanced machines used to solve computability
- It doesn't have a memory, therefore, only having the ability to **recognize** the basic class of languages: regular languages
- There are two types of finite automata: Deterministic and Nondeterministic

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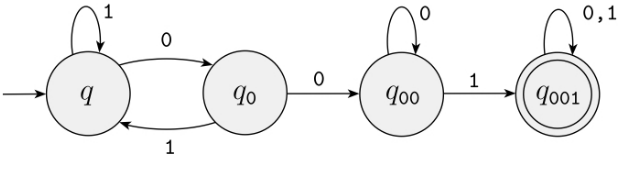

Figure: DFA Example

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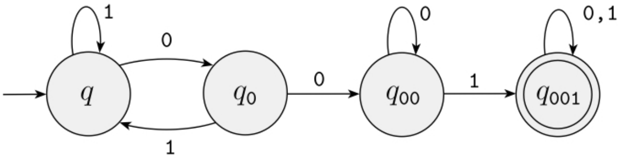

Figure: DFA Example

- States: Possible positions of the machine (e.g., q, q_0, q_{00}, q_{001}).
- Transitions: Arrows indicating state changes based on input (e.g., $q \rightarrow q_0$ on input 0).
- Start State: Initial state, marked by an incoming arrow (e.g., q).
- Accepting States: States where input is accepted, denoted by double circles (e.g., q_{001}).

DFA Definition

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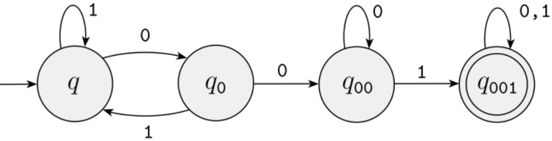

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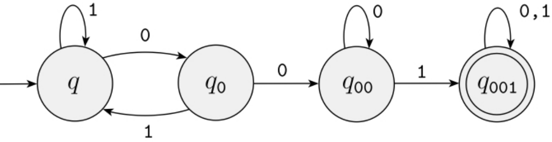

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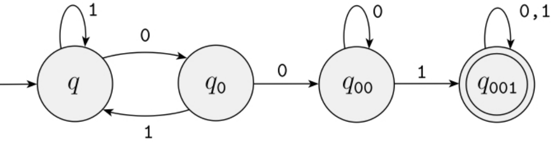
Definition of a DFA

A finite automata is a 5-tuple, which means it is defined by five components:

$$M = (Q, \Sigma, \delta, q, F)$$

DFA Definition



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Definition of a DFA

A finite automata is a 5-tuple, which means it is defined by five components:

$$M = (Q, \Sigma, \delta, q, F)$$

1. Q is a finite set of states,
2. Σ is the input alphabet (the symbols the DFA can read),
3. $\delta : Q \times \Sigma \to (Q)$ is the transition function, which tells the DFA how to move from one state to another,
4. $q \in Q$ is the start state, where the DFA begins, and
5. $F \subseteq Q$ is the set of accepting states.

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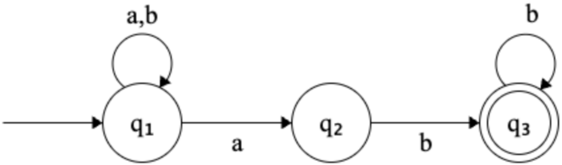

Figure: NFA Example

What is a NFA?

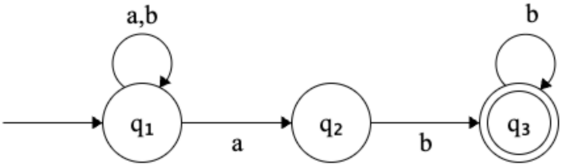

Figure: NFA Example

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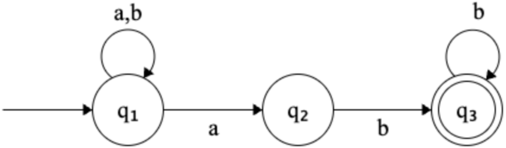

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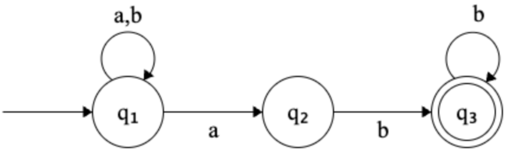

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Definition of a NFA

The definition of a nondeterministic finite automata is a 5-tuple, which means it is defined by five components:

$$M = (Q, \Sigma, \delta, q_1, F)$$

NFA Definition

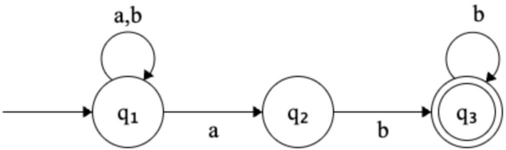

Figure: NFA Example

Definition of a NFA

The definition of a nondeterministic finite automata is a 5-tuple, which means it is defined by five components:

$$M = (Q, \Sigma, \delta, q_1, F)$$

1. Q is a finite set of states,
2. Σ is the input alphabet (the symbols the NFA can read),
3. $\delta : Q \times \Sigma\epsilon \rightarrow P(Q)$ is the transition function
4. $q_1 \in Q$ is the start state, where the NFA begins, and
5. $F \subseteq Q$ is the set of accepting states.

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Definition of Equivalence

Equivalence means that two different types of machines (like a DFA and an NFA) can recognize exactly the same set of strings or languages.

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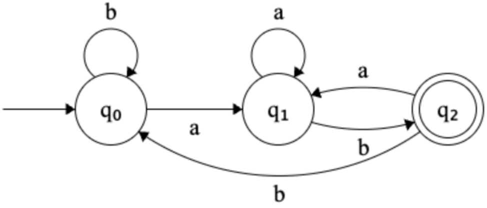
Equivalence means that two different types of machines (like a DFA and an NFA) can recognize exactly the same set of strings or languages.

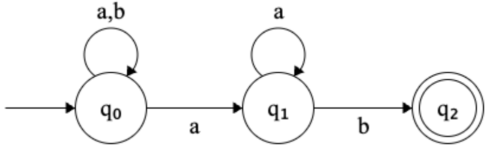
States: Both consist of states that process input symbols, with a start and one or more accepting states.

Pathways: DFA : Exactly one path per state-symbol pair (like a single-lane road).
NFA : Multiple or zero paths per state-symbol pair (like a choose-your-own-adventure book).

Acceptance: DFA: Accepts if a single path leads to an accepting state.
NFA: Accepts if *any* path leads to an accepting state.

Example of an Equivalent DFA and NFA





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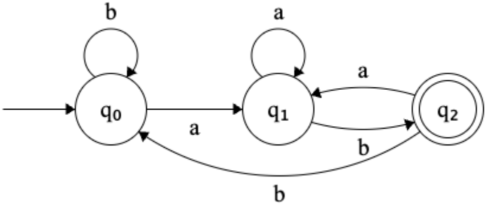

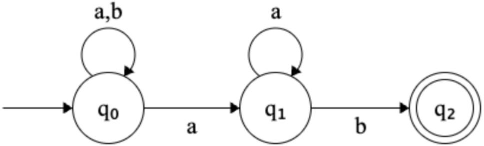

The set of all strings over the alphabet a,b that contain the substring "ab"

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- All these may seem very abstract, but they're the fundamentals to understanding larger issues in computer science
- Finite automata and other machines: programming languages and parsing
- Finally, the theory of computation allows us to determine whether a problem is solvable or unsolvable

A Brief Introduction to TOC

This is Sherri's section on an introduction to TOC: Watch Video

References

Michael Sisper (2013)
Understanding the Theory of Computation 3rd ed.

Thank you to...

Katherine Taylor for guidance
Our wonderful research group
Everyone listening!