Donuts in Different Dimensions

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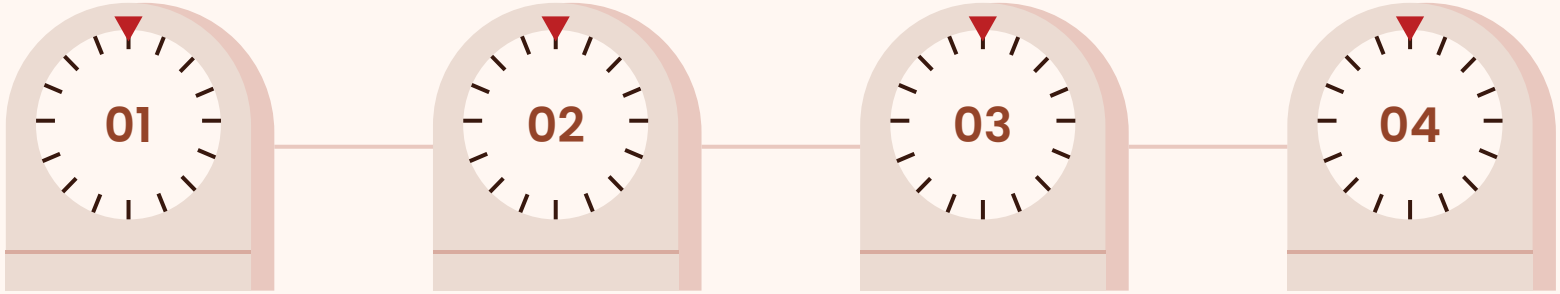


01

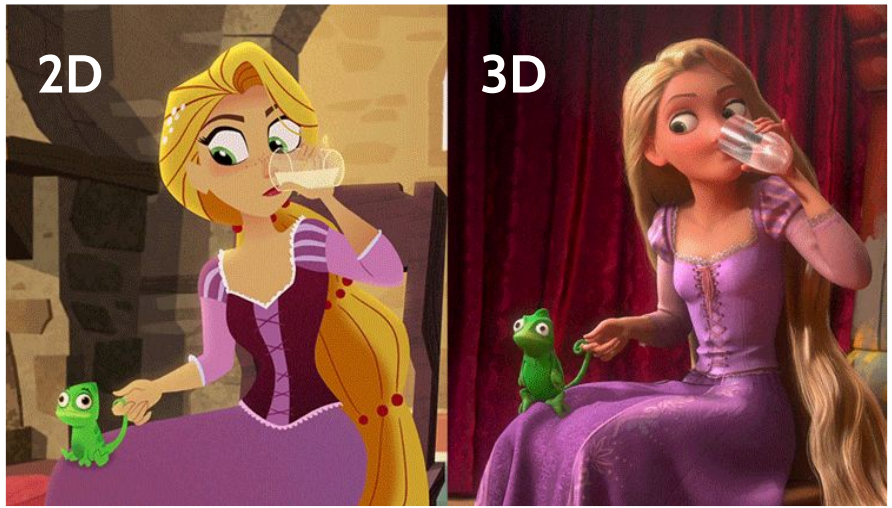
What *is* a dimension?

1.1

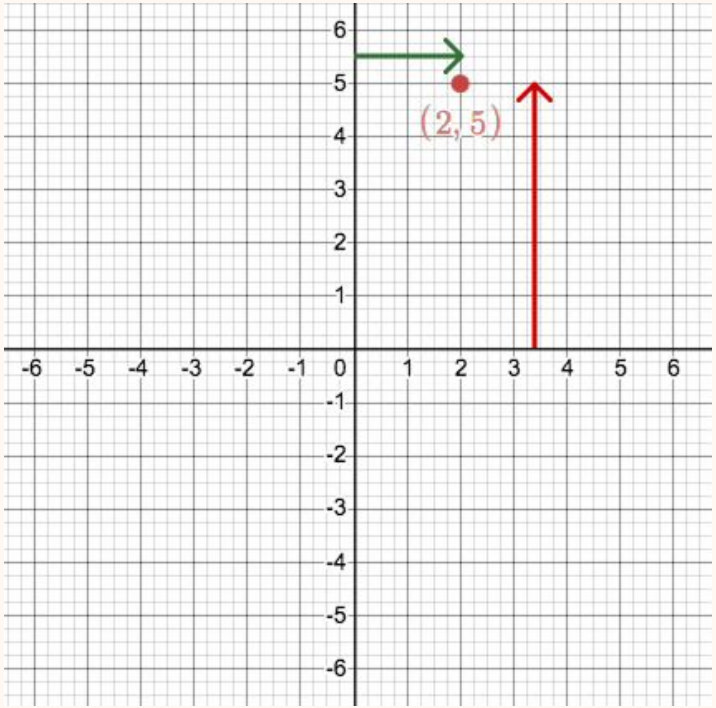
★ We often hear about **2D** and **3D animation**

But what does this really mean?

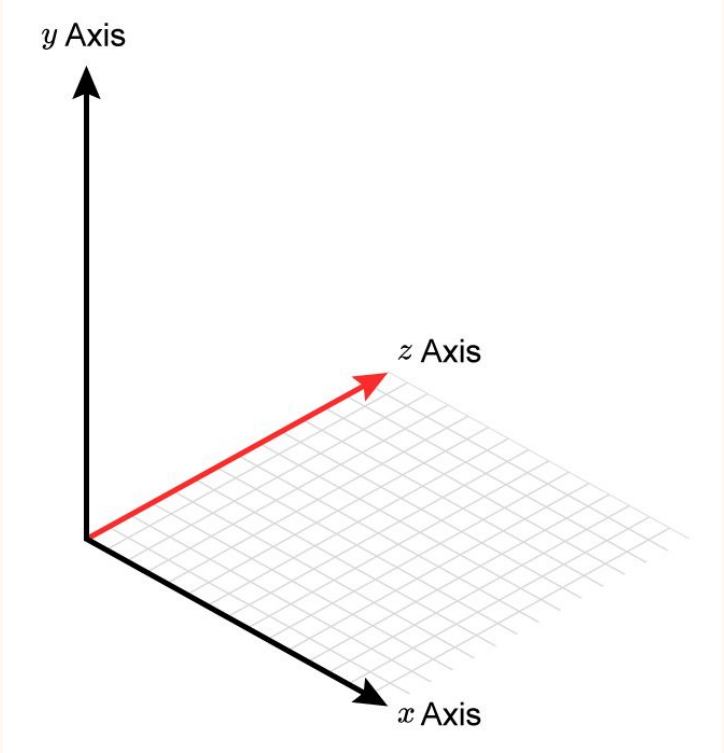
What's the difference?



1.2 A dimension is just a direction you can move.



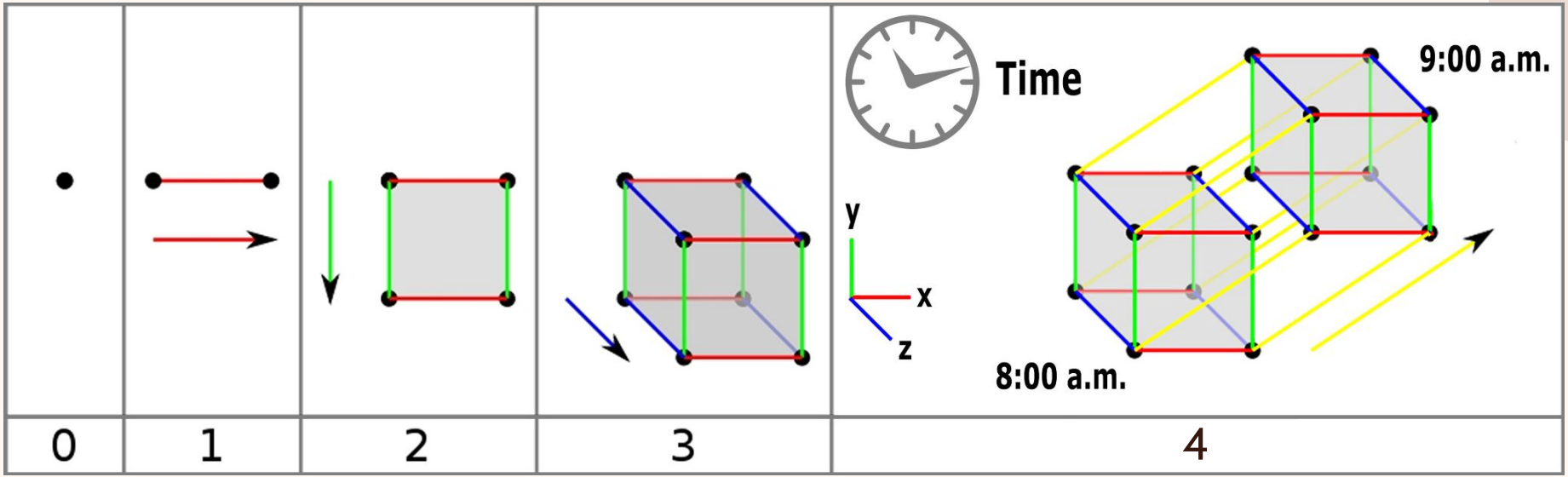
Source: https://sites.ps.uci.edu/mathceo/meeting-2/picturing-sequences



Source: https://mammothmemory.net/maths/graphs/graphs/z-axis.html

1.3

The more Dimensions, the more Directions



Sources: https://storymaps.arcgis.com/stories/64874fecff904d67baed476fcf8755d1, https://johncarlosbaez.wordpress.com/2013/06/04/symmetry-and-the-fourth-dimension-part-10/

1.4

★ We won't be exploring donuts in **0D** and **1D**

WHY NOT?

Shapes in 0D and 1D look something like this, which we'd consider to be "trivial"

0D
1 point, no movement

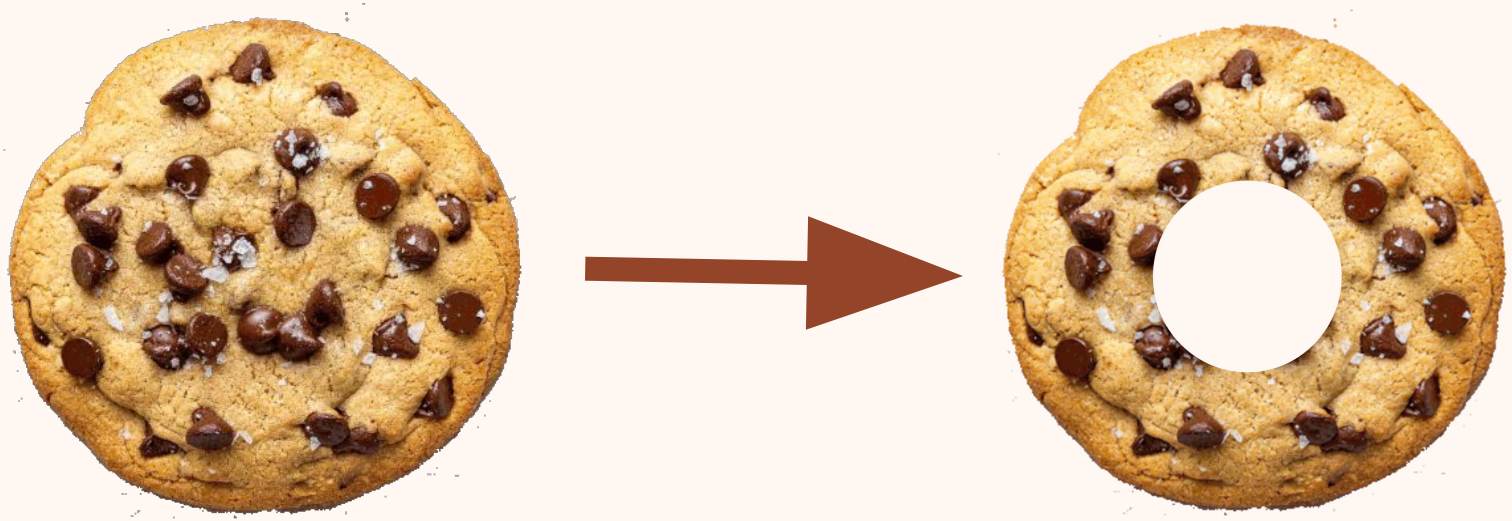
1D
Circle, you can move around it, but only in one direction

02

2D Donuts

2.1

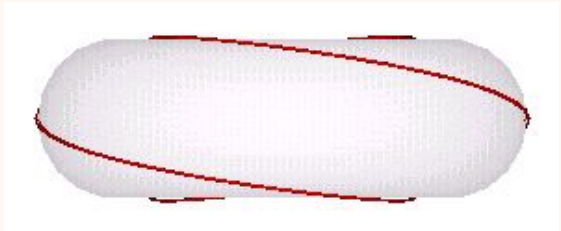
You can’t have a flat donut, it would just be a cookie.



2.2

THE TORUS

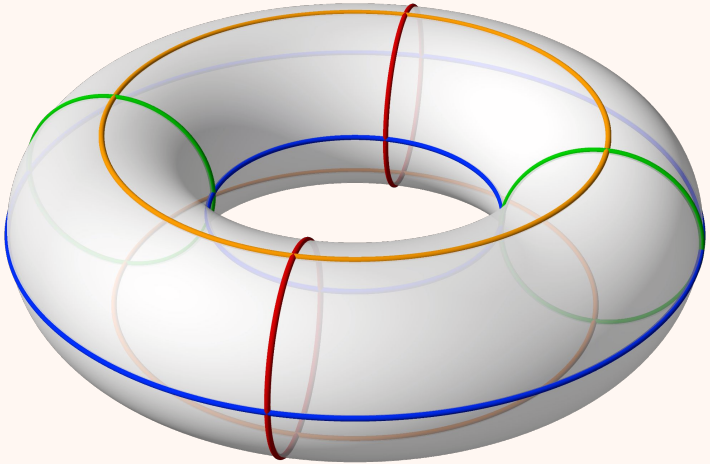
Source http://www.giftofmath.com/pacman.html



Source http://www.rwgrayprojects.com/Lynn/torusm01/tm01.html:

2.3

THE TORUS



2D Surface

Source: https://commons.wikimedia.org/wiki/File:Tesseract_torus.png

Solid Torus

2.4

MANIFOLDS

★ A **manifold** is a shape that looks flat when you zoom in close enough.

- They can exist in several dimensions.
  - The **glaze** on a donut, excluding the donut itself, is a **2-manifold**.
  - An **n-dimensional** manifold looks like **n-dimensional** space when you zoom in.
    - Ex. **2D manifold** looks like **2D space**

The earth is *not* flat, it just <u>feels</u> that way.

2.5

2-MANIFOLDS

★ 2-manifolds are also called **surfaces**.

The glaze on a donut is a 2-manifold

- They don't have boundaries (glaze wraps around the **entire** donut)
    - Every point on the glaze has a **neighborhood** around it.

03

3D Donuts

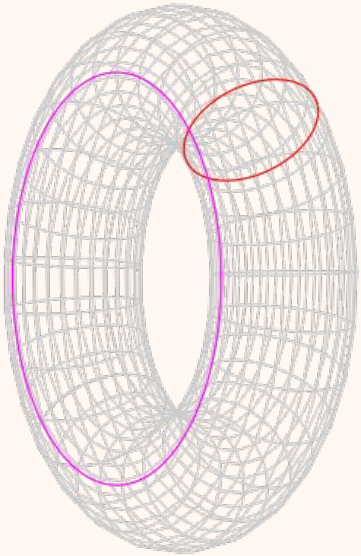
3.1

3-MANIFOLDS

★ A 3-manifold looks like regular 3D space when you zoom in.

➔ If you lived inside it, everything would feel normal: you could move up, down, left, right, forward, and backward.

Source: https://verse-and-dimensions.fandom.com/wiki/Solid_torus

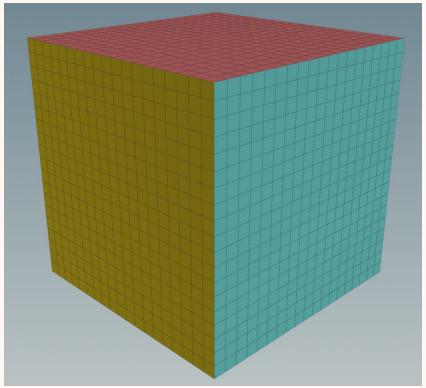


Source: https://en.wikipedia.org/wiki/Torus#/media/File:Torus_cycles.svg

3.2

ISOTOPY

★ Isotopy is a way of transforming one shape into another without tearing or gluing parts together.



A cube deforming into a sphere

Source: https://www.artstation.com/blogs/briz/oKpM/perfectly-remapping-a-cube-to-a-sphere-houdini

A coffee mug deforming into a solid torus

Source: https://upload.wikimedia.org/wikipedia/commons/2/26/Mug_and_Torus_morph.gif

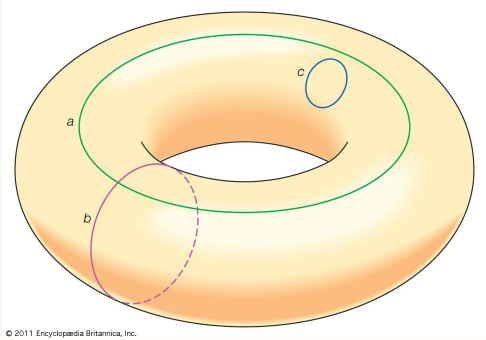
3.3

HOMEOMORPHISM

★ Around any point ***n***, there's a **neighborhood of points** that can be mapped continuously and reversibly to flat n-dimensional space. This mapping is called a **homeomorphism**.

➔ Example **(2D)**: A small patch on a 2-manifold maps to a flat **2D plane** (like a sheet of paper).

➔ Example **(3D)**: A chunk inside a 3-manifold maps to **regular 3D space** (like the space around you).



Source: https://www.britannica.com/science/topology/Homeomorphism

3.4

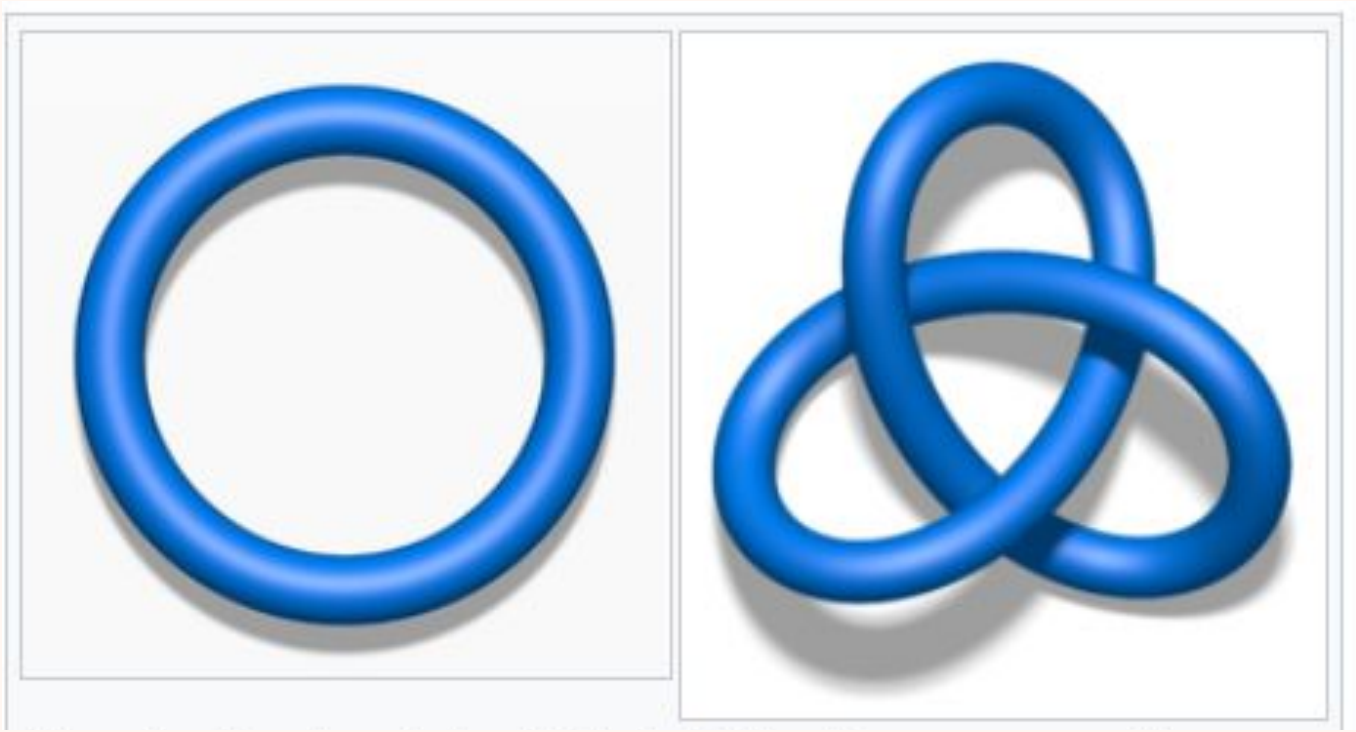
ISOTOPY VS. HOMEOMORPHISM

- **Homeomorphism**: Two shapes made of flexible clay, using math you can try to figure out whether you *could* reshape one into the other.

- **Isotopy**: You actually mold one into the other *with your hands*, step by step, without tearing or fusing.

★ **All isotopic shapes are homeomorphic. Not all homeomorphic shapes are isotopic.**

3.5



04

Why do we care?

4.1 DONUTS TO UNDERSTAND MATH

- Everything we've explored so far are all fundamental parts of a branch of math called **Topology**.

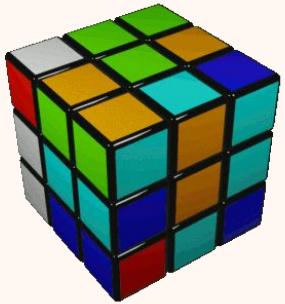
Topology is the study of the properties of shapes and spaces that remain unchanged under continuous deformations, such as stretching or bending, but not tearing or cutting.

4.2 TOPOLOGY IN THE WORLD

➔ **DNA**: understand how DNA strands twist and knot

- Crucial in biology for processes like **replication** and **repair**.

➔ **Rubik's Cube**: Help in analyzing the cube's rotational symmetries and solving algorithms.



Q & A

Any questions?