Graph Theory and its Real Life Applications

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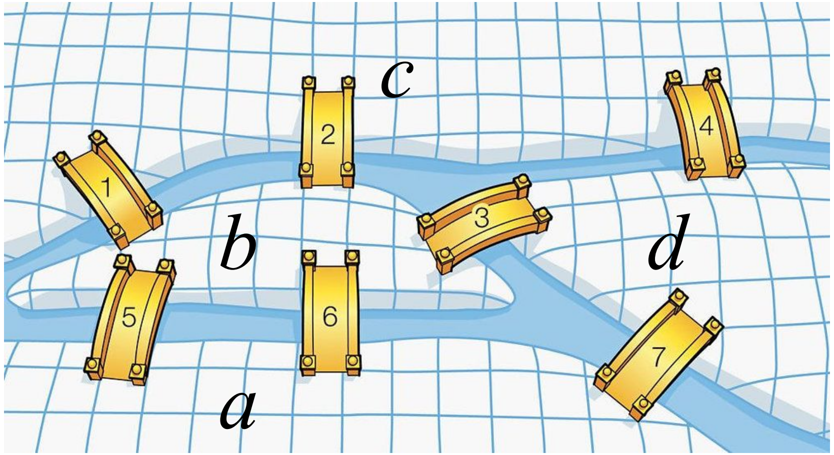
History of Graph Theory

- Königsberg is made up of 4 landmasses and 7 bridges
- People wondered if one could walk across every bridge exactly once
- Leonhard Euler tried to solve it
- He drew out the city in a more simple way

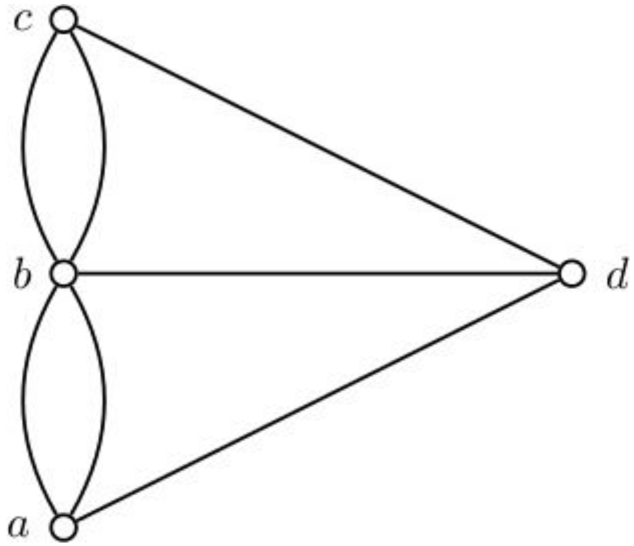


The City of Königsberg

Leonhard Euler

Discoveries

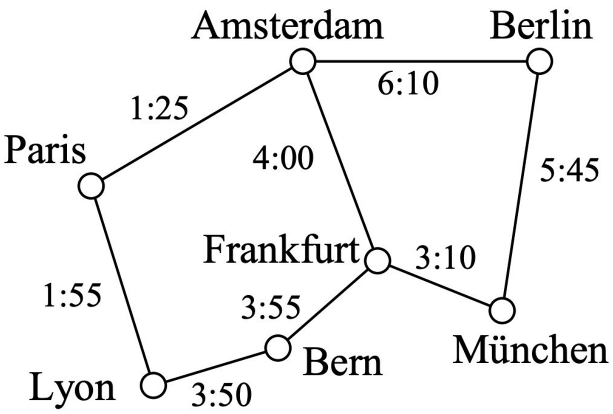
- It is impossible to walk across every bridge only once
- If more than 2 vertices had an odd number of bridges connecting to it, it was impossible
- If there are 2 vertices with an odd number of bridges, then it would be possible **only** if we started from one of those two vertices.
- If there were no landmasses with an odd number of bridges, it would always be possible.



Simple Version of Königsberg

Why Is This So Important?

- Laid the foundation for Graph Theory
- Studies relationships and connections between objects
- Helps with finding efficient routes (GPS, Google Maps, etc.)
- Important in social media (suggested accounts, followers, etc.)



Example of a train network

Basic Definitions

Vertex

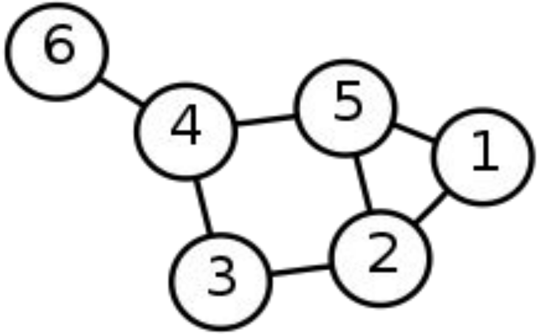
Basic building block of graphs, also called a node. It is the most important component of a graphs structure.

Edge

The other fundamental piece in a graph that connects vertices together or to themselves.

Degree

The number of edges connecting to a vertex. The vertices that are connected to make these edges are called neighbor vertices.



Vertex 1 has a degree of 2.

This graph has a order of 6 and a size of 7.

Order of a Graph

The number of vertices in a graph. This is noted as V(G).

Size of a Graph

The number of edges in a graph. This is noted as E(G)

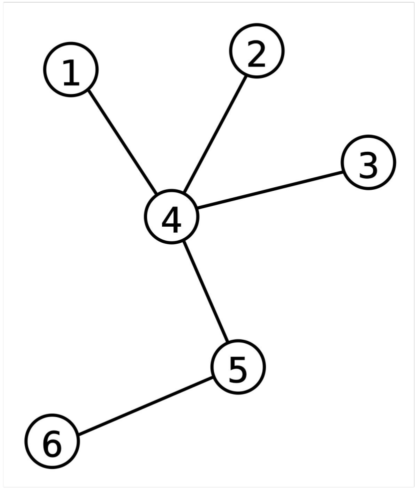
Topics in the Paper

1. Graph Traversal (What are the different ways to move across a graph?)
2. Connected Graphs (What does this allow us to do?)
3. Common types of Graphs
4. Graph Traversability (Related to Königsberg Bridge Problem)
5. Shortest Path Problem
6. Minimum Spanning Tree Problem

Trees, Weighted Graphs, and Minimum Spanning Trees

Tree: A Connected Acyclic Graph.

Acyclic: A Graph that does not have any cycles.

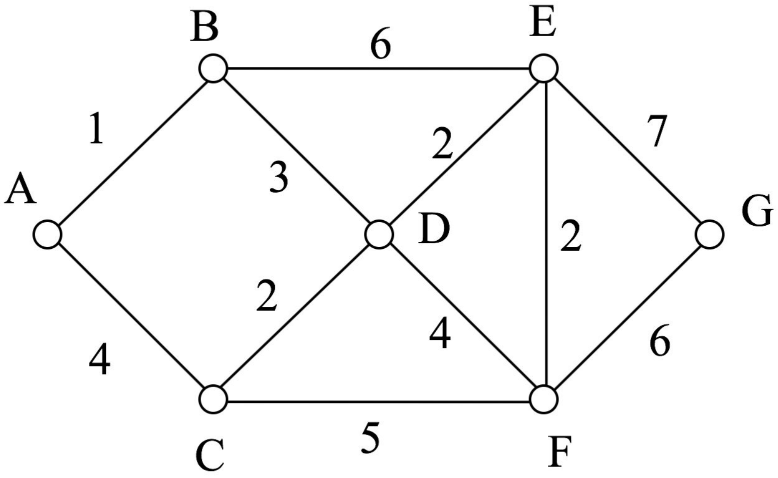


Example: On the graph on the right we can observe each edge of the graph. Notice that if we remove any edge of the graph it will result in a incomplete graph. Meaning each edge is a bridge. Thus a graph can only be a tree if all edges are a bridge.

Trees, Weighted Graphs, and Minimum Spanning Trees

Weighted Graphs: A weighted graph is a graph were all edges are assigned a specific value usually depending on a certain situation.

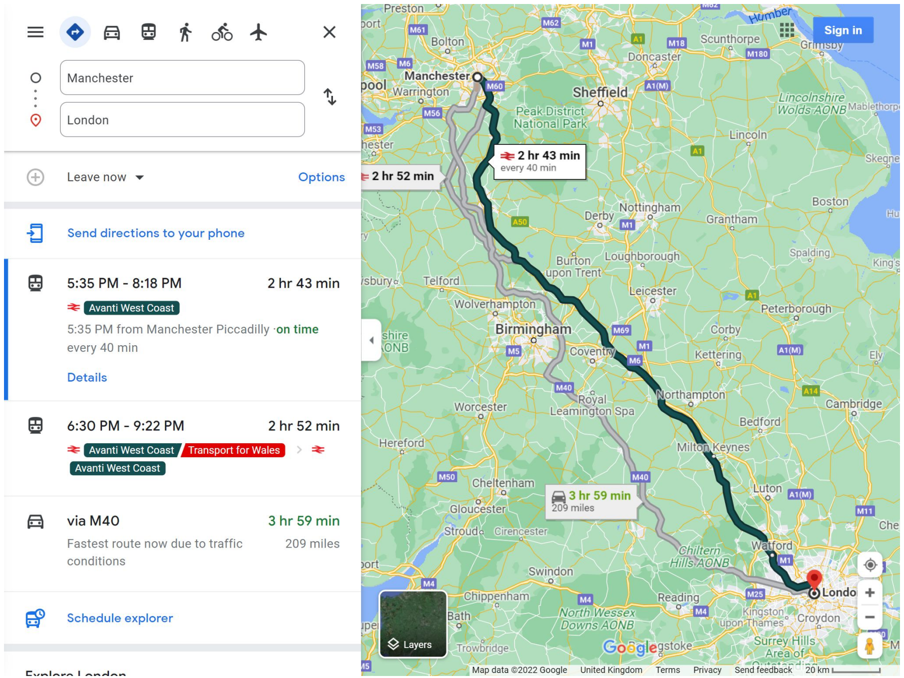
G:



Example: On the left this is a weighted graph with all edges assigned a value. We can add all the numbers of the graph and it will give us the equation 6 + 1 + 7 + 2 + 3 + 2 + 4 + 5 + 4 + 6 + 2 = G, meaning graph G = 42.

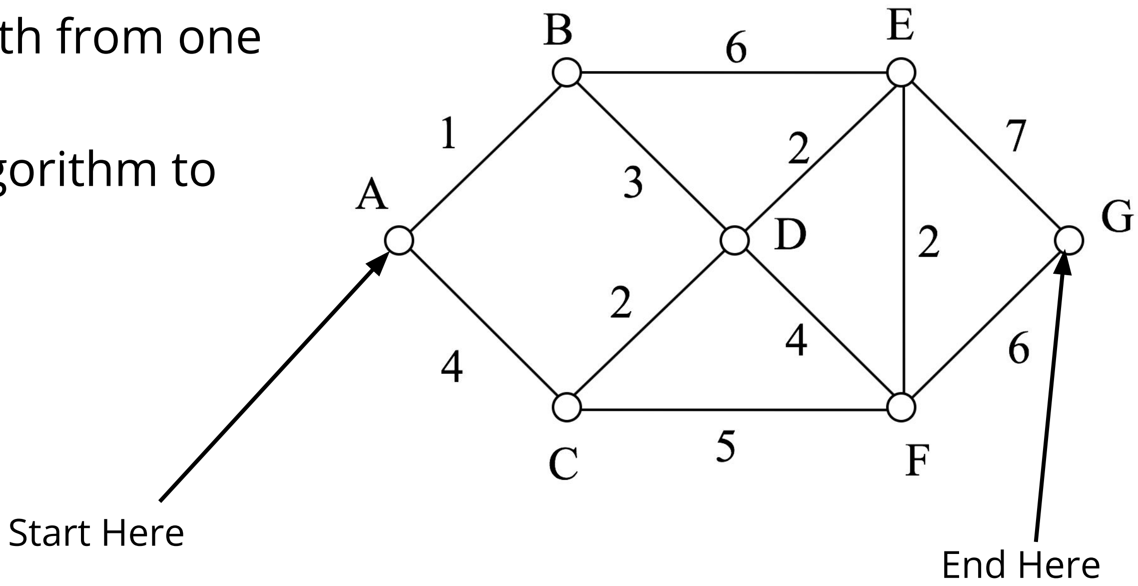
Why Are Weighted Graphs Important?

- Representative of Real Life
- Important to find the most efficient route to other place
- How to find the shortest path?



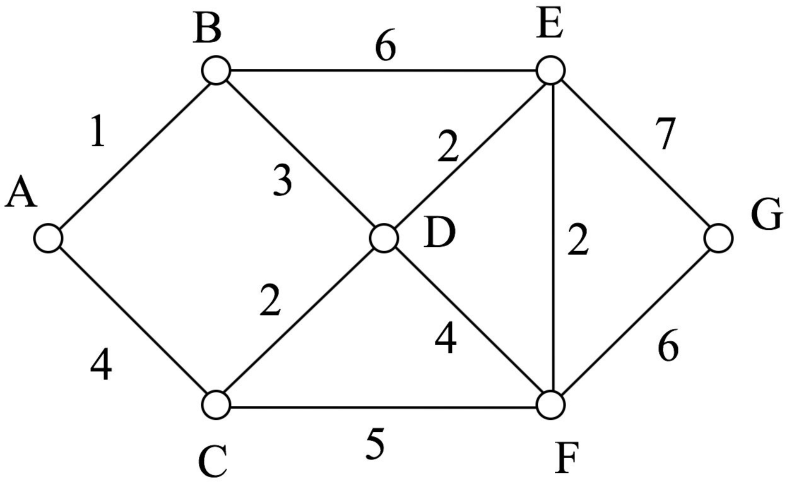
Shortest Path Problem

- Find the shortest path from one vertex to another
- We use Dijkstra’s Algorithm to find it
- Efficient is better



Trees, Weighted Graphs, and Minimum Spanning Trees

Minimum Spanning Tree: A tree within a weighted graph that has the least amount of value without repeating any edges to get one point to another.

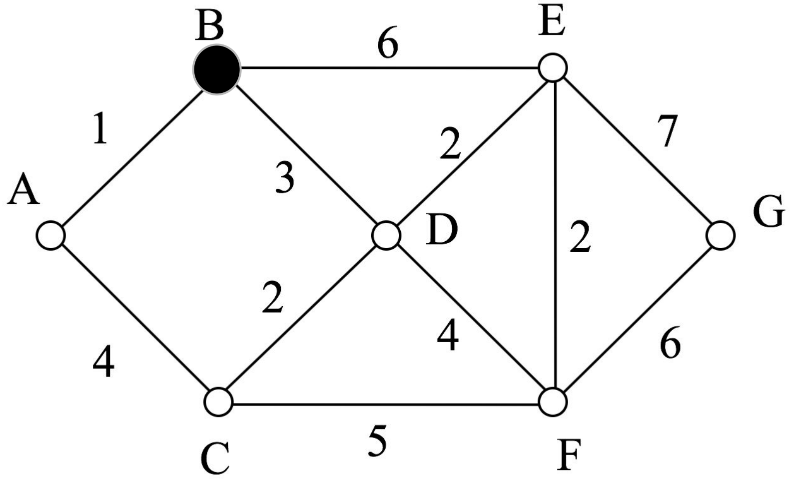


Example: Back to the equation from the last slide, this is very important when trying to find the path with the least value. Let’s say we want to get to every vertex with the least amount of value, how would we do this?

Step 1 & 2

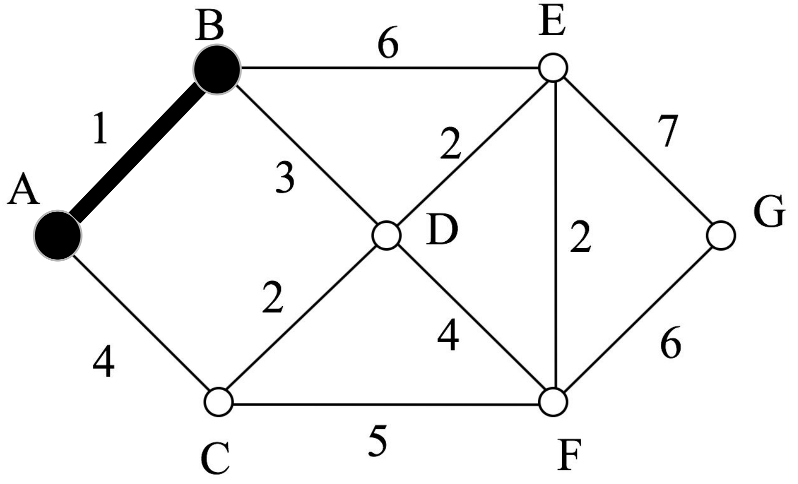
- Start with a weighted graph
- Choose a starting vertex

Let's start with vertex B.



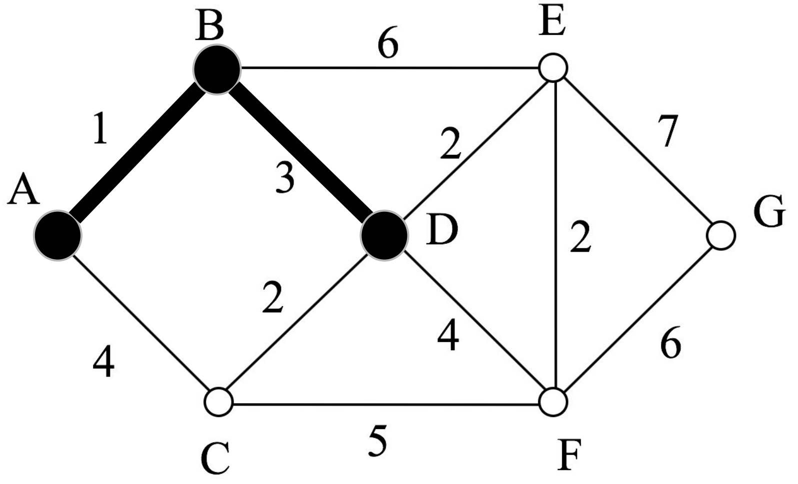
Step 3

- Choose the edge of minimum weight *of the vertex* and add it to the tree
- We choose the highlighted edge because it has the least weight between all of vertex *B*'s edges.



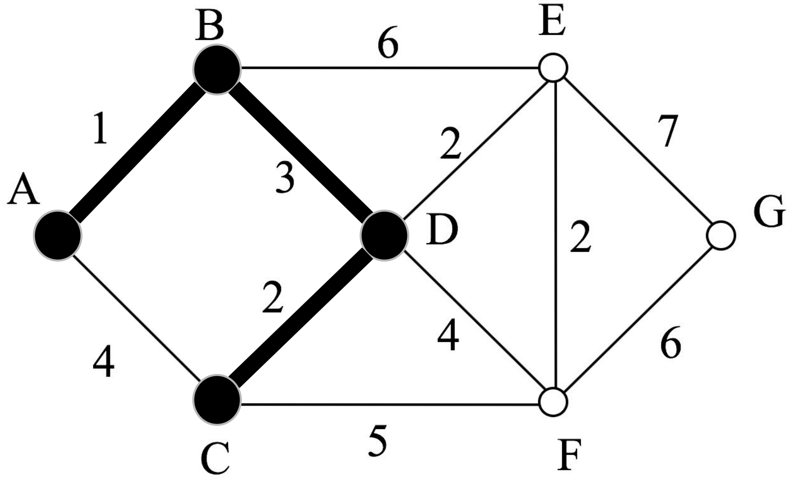
Step 4

- Add the next smallest edge to the tree
- The edge from B-D has the least weight, so we add it to the tree.



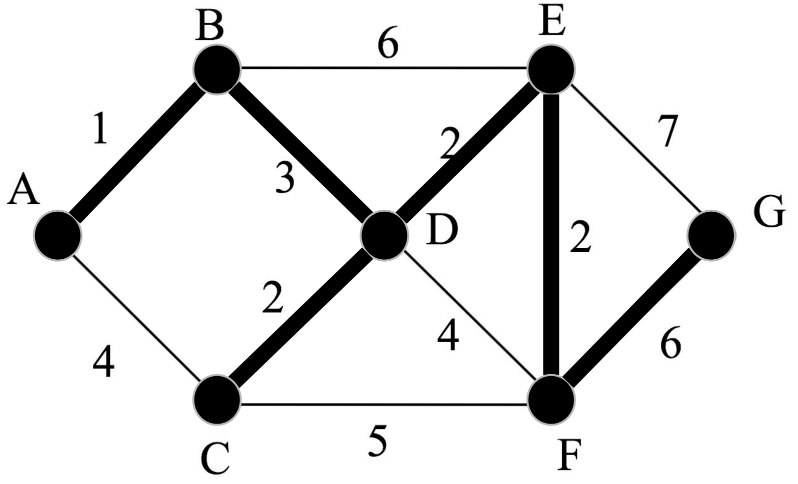
Step 5

- Add the next smallest edge, unless that edge would create a cycle, which would not make it a tree.
- If there are 2 choices, choose any.



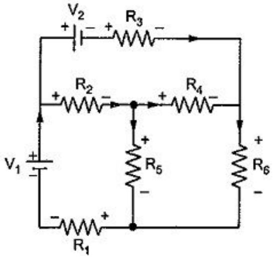
Step 6

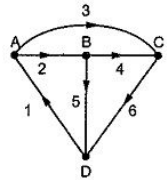
- Repeat until you have the minimum spanning tree

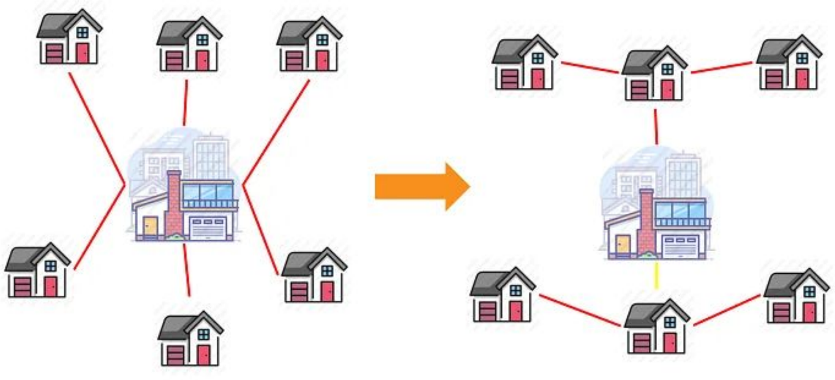


How Does This Apply to Real Life?

- **Minimum Spanning Tree Problem**
  - Electrical Networks
  - Telecommunication networks
  - GPS Routes (especially helpful with multiple stops)







Naïve telecommunication routing, generates new connection line for each customer, which leads to huge cost.

Minimum Spanning Tree Routing generates connection line in less cost with high efficiency.