

# Disarray in the Tribarray

A Tribsploration

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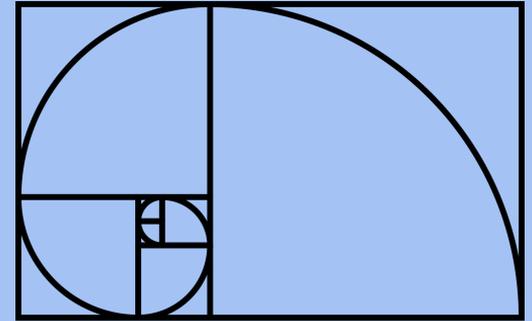
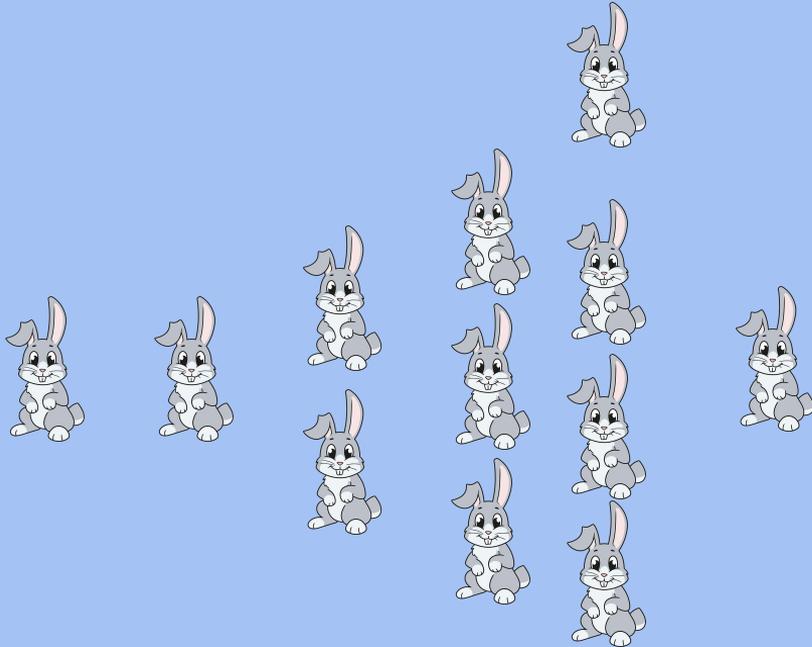
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# Fibonacci Sequence

The Fibonacci sequence starts with 0, 1, and each successive term is the sum of the two previous terms.

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ...



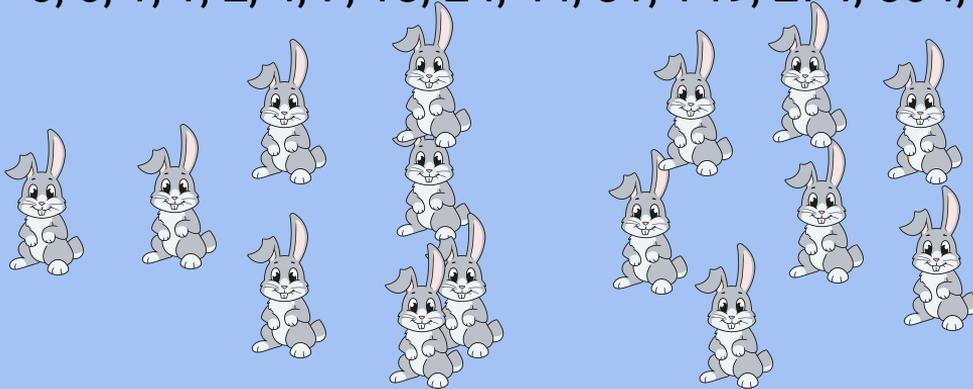
# Tribonacci Sequence

The Tribonacci numbers  $T_n$  start with  $T_0 = 0$ ,  $T_1 = 0$ ,  $T_2 = 1$ . Each Tribonacci number after is calculated by summing the three previous numbers:

$$T_n = T_{n-1} + T_{n-2} + T_{n-3}.$$

This makes the Tribonacci sequence:

0, 0, 1, 1, 2, 4, 7, 13, 24, 44, 81, 149, 274, 504, 927, 1705, ...



# Tribonacci Representation

The Tribonacci representation of an integer  $k$  is its expression as a sum of positive Tribonacci numbers.

- Each Tribonacci number can be used only once.
- No 3 consecutive Tribonacci numbers.

It can be viewed as expressing the number in the Tribonacci base.

Example:

1, **2**, 4, **7**, **13**, 24, 44, 81, 149...

$$22 = 2 + 7 + 13$$

The Tribonacci representation of 22 is 11010.

Every natural number has a unique Tribonacci representation.

# Tribonacci Base

We can represent the Tribonacci representation of a number in the Tribonacci base similarly to how we express numbers in numerical bases.

Example:

$$53 = 2 + 7 + 44$$

...	0	1	0	0	1	0	1	0
...	81	44	24	13	7	4	2	1

The Tribonacci representation of 53 is 1001010.

# Generating the Tribonacci Representation

We find the Tribonacci representation of a number through a greedy algorithm.



- Find the largest Tribonacci number that is less than or equal to the number and subtract it.
- Repeat the process until we reach 0.

This method ensures that no three consecutive Tribonacci numbers are used.

Example:

1, **2**, 4, **7**, 13, 24, **44**, 81, 149...

$$53 - 44 = 9 \rightarrow 9 - 7 = 2 \rightarrow 2 - 2 = 0$$

We then express the number in the Tribonacci base.

# Tribonacci Successor

Suppose an integer  $n$  has Tribonacci representation:

$$n = T_{i_1} + \cdots + T_{i_k}$$

Then we define the Tribonacci successor of  $n$  as  $\text{out}(n)$ , where

$$\text{out}(n) = T_{i_1+1} + \cdots + T_{i_k+1}.$$

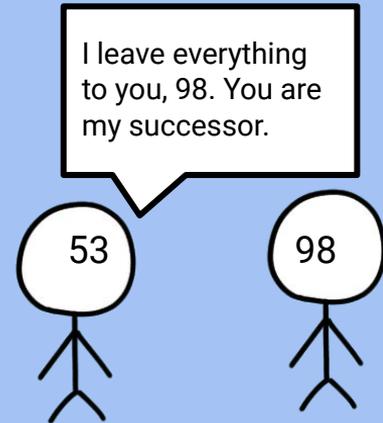
Example:

0, 0, 1, 1, **2**, 4, **7**, 13, 24, **44**, 81, 149...

0, 0, 1, 1, 2, **4**, 7, **13**, 24, 44, **81**, 149...

$53 = 2 + 7 + 44 \rightarrow 1001010$

$\text{out}(53) = 4 + 13 + 81 = 98 \rightarrow 10010100$



# Tribonacci Constant

The Tribonacci numbers grow approximately as a geometric series with the ratio equal to the Tribonacci constant ( $\alpha$ ). Thus,

$$\text{out}(n) \approx \alpha^n$$

The Tribonacci constant is approximately 1.83929.

# A Fun Use of the Tribonacci Successor

Picture this: you are in a life-threatening emergency where you have to convert between Kelvin and Degrees Rankine. But you don't have a calculator!

Don't you worry! If you take a temperature  $n$  in degrees Kelvin, then  $\text{out}(n)$  is close to the temperature in degrees Rankine. For example, take the temperature 300 degrees Kelvin.

$$300 = 274 + 24 + 2 = 1000100010$$

$$\text{out}(300) = 504 + 44 + 4 = 10001000100 = 552$$

And 300 degrees Kelvin is equal to 540 degrees Rankine.







# TRIBvestigating Past the Wall: Precolumns

We extend the Tribarray to the left, by using the rule that in every row each number is the sum of the previous three numbers. We call the 0th column *the wall*, column -1, the *seed*, and column -2, the *pre-seed*. The normal Tribarray can also be called the *garden*.

-2	-1	0	1	2	3	4	5	6	7	
0	0	1	1	2	4	7	13	24	44	...
0	1	2	3	6	11	20	37	68	125	...
1	1	3	5	9	17	31	57	105	193	...
1	2	5	8	15	28	51	94	173	318	...
1	3	6	10	19	35	64	118	217	399	...
2	3	7	12	22	41	75	138	254	467	...
2	4	8	14	26	48	88	162	298	548	...
...	...	...	...	...	...	...	...	...	...	...





# TRIBtracting terms: Difference Sequences

For any row in the Tribarray, there exists a difference sequence, containing the difference between consecutive terms.

Example:

**WHAT'S THE  
DIFFERENCE?**

Row 1: 1, 2, 4, 7, 13, 24, 44, 81

The difference sequence is 1, 2, 3, 6, 11, 20, 37, which is also a row in the Tribarray (it's row 2).

All difference sequences of Tribarray rows are other rows.

# Inverse Difference Sequences

What if we go the other way around? Given a difference sequence that is an extraTrib, can we find the original extraTrib?

Sometimes. Most rows are the difference sequence of another row, but not all of them.

We have a simple method of inverting a row, which involves multiplying the Tribonacci representations by 11.

There are no cycles, so you if you keep taking the inverse of a row you will never get back to where you started.

# Different Types of Tribonacci Representations

Not all representations are created equal:

Canonical: contains only 0 and 1, does not have three 1s in a row (e.g. 100001101)

Non-canonical: contains only 0 and 1, may contain three 1s in a row (e.g. 11111)

Improper: may contain digits other than 0 and 1 (e.g. 2022)



# The TRI(B)ck to Canonization

We created a procedure to convert an improper representation into a canonical representation through repeated carrying.

Using this, we can easily multiply the Tribonacci representations of extraTribonacci numbers to get improper representations, then convert those to canonical representations to show that multiples of extraTribonacci numbers are also extraTribonacci numbers.



# TR(IB)ying to get close: Approximations



Using the following algorithm we can approximate any term in the Tribarray.

First we approximate the corresponding first column term with the ratio  $\frac{r\alpha}{\alpha-1}$  where  $r$  is the row number.

To move to the right we repeatedly apply the function  $\text{out}(n)$ , which can be approximated by  $n \cdot \alpha$ .

Generally, a term in row  $r$  and column  $c$  is approximated by  $\frac{r\alpha^c}{\alpha-1}$ .

# MulTRIBples of the Tribonacci Sequence

All multiples of rows of the Tribarray show up as other rows in the Tribarray.

For example, row 1 of the Tribarray:

1, 2, 4, 7, 13, 24, 44, 81, 149, 274, 504, ...

Multiplying each of these terms by 4 gives us

4, 8, 16, 28, 52, 96, 176, 324, 596, 1096, 2016, ...

These terms show up in row 28, which is

176, 324, 596, 1096, 2016, ...

## Extending an extraFib to the left

When we extend an extraFib to the left, we get

..., -21, 13, -8, 5, -3, 2, -1, 1, 0, 1, 1, 2, 3, 5, 8, 13, 21, ...

It's the same extraFib, but with alternating signs!

We don't always get the *same* sequence when extending to the left, but it's always an extraFib.



# Now for Some TRIBvia!

Extending an extraFib to the left gives us another extraFib with alternating signs, but what does an extraTrib extended to the left look like?

## A DisTRIBing Answer...

Nothing nice, just disarray!

When we extend an extraTrib to the left, it looks like this:

...0, 56, -47, 9, 18, -20, 7, 5, -8, 4, 1, -3, 2, 0, -1, 1, 0, 0, 1, ...

No alternating signs here.

Extending it further just makes it worse:

...-271, 421, -206, -56, 159, -103, 0, 56, -47...



# Some Generalizations

The extraFibs extended left alternate in signs (no 2 consecutive numbers have the same sign)

Extending the extraTribonacci left, no 3 consecutive numbers have the same sign.

We also explored other properties and applications of the Tribonacci Sequence and the Tribarray.

# ConTRIButors

Thank you to Tanya Khovanova for mentoring and supporting our group.

Thank you to PRIMES STEP, MIT, and Slava Gerovitch for the opportunity.

Thank you to our parents who drove us to MIT every week.

# Results

- Multiples of row
- Tribonacci word stuff
- ~~All the messy computational stuff~~
- Difference sequences + inversions
- Approximation of successor
- Approximation of any term in the array
- Reversal
- Going to the left/precolumns
- Tribternary(the b is silent)

# Tribonacci puns

- DisTRIButions
- Multiples = MulTRIBples
- ConTRIBution
- TRIBvia
- DiscTRIBtions
- Do you have a hurT RIB?
- nuTRIBtion
- T.R.I.B.:To Read It Backwards

# Ideas

~~Picture: Crossed out bike, have trike~~

Triangles?

Kelvin to rankine

Attribute

Contribute

~~Canonization: show a cannon~~

Canon in trib major