1) Background Information
   a) Defining a Graph
   b) Defining a Matrix
2) Exploring Specific Graphs and Matrices
   a) Directed Graph
   b) Weighted Graph
   c) Incidence Matrix
   d) Transition Matrix
3) Exploring Applications of Graph Theory
   a) Dijkstra Algorithm
   b) PageRank
But first… why might we care about Graph Theory?
Preview of Important Applications of Graph Theory

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**Dijkstra Algorithm** and **PageRank**.

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- **Dijkstra Algorithm** is used to help find the shortest path from one point to another.
- **PageRank** is used to determine the most relevant webpages when a user makes a search in a search engine.
- Both of these applications are used by us everyday without even realizing it.
What is a Graph?

Graph: a collection of edges and vertices
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An example of a graph with a labelled edge and vertex
What is a Matrix?

- **Matrix**: a set of numbers arranged in rows and columns so as to form a rectangular array
- Matrices often provide valuable information about graphs.
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An example of a matrix with two labelled entries:
Weighted Graph

**Weighted Graph:** A graph where each edge is assigned a certain value or “weight”
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An example of a weighted graph with a labelled weighted edge: An example of a edge between vertex 2 and vertex 4 with a weight of 2.

An example of a weighted graph with a labelled weighted edge.
Directed Graph

- **Directed Graph**: A graph with directed edges
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  - Each edge has a starting vertex and a destination vertex.
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An example of a directed edge leaving vertex 2 and arriving at vertex 4.

A directed graph with a labelled directed edge.
Incidence Matrix

**Incidence Matrix:**

- The matrix of a directed graph made up of 1’s, -1’s, and 0’s
  - 1 = leaving a vertex
  - -1 = arriving at a vertex
  - 0 = vertex is not involved
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![Incidence Matrix Diagram]

An incidence matrix to represent the graph above.

This entry of -1 represents that the edge between vertex 3 and 1 is leaving vertex 3 and arriving at vertex 1.
Transition Matrix:

- A matrix based on the weights assigned to each edge in a directed weighted graph.
- Rows represent the weight of the edges arriving at a vertex.
- Columns represent the weight of the edges departing from a vertex.

A transition matrix based on the graph above:

\[
\begin{bmatrix}
0 & 2 \\
1 & 0 \\
\end{bmatrix}
\]

This is an example of a transition matrix created based on a directed weighted graph.
So how can all of this information be applied?
1st Application of Graph Theory: Dijkstra Algorithm

- The **Dijkstra Algorithm** is used to find the shortest path from the ‘source node/vertex’ to all the adjacent/surrounding nodes.
1st Application of Graph Theory: Dijkstra Algorithm

- The **Dijkstra Algorithm** is used to find the shortest path from the ‘source node/vertex’ to all the adjacent/surrounding nodes.
- This method is still currently used by navigational apps, including Google Maps, Apple Maps, and more.
Representation of Dijkstra Algorithm

- Dijkstra Algorithm can be used on a weighted or unweighted graph, it can also be used on a directed or undirected graph
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2nd Application of Graph Theory: PageRank System

- **PageRank** determines the relevance of a webpage based on how many other webpages link back to that particular webpage.
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This system was adopted in place of a text based ranking system, which was used up until the 1990s.
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An example of what page ranking may look like. Because all the other web pages link to webpage 3, webpage 3 would be considered the most important.
Representation of PageRank

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- An arrow moving away from a webpage shows that the webpage links to the webpage that the arrow is pointed toward.
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This is an example of the connections between website 1 and 4. The graph shows that website 1 links to 4 and website 4 links to website 1.
Page Rank Can Also Be Represented by a Directed Weighted Graph and Transition Matrix as Shown Below:

This is an example of directed weighted graph that shows the links between 4 websites.
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- The weight to each arrow leaving the same web page is equal and the weights of all the arrows leaving the same web pages add up to 1.
Page Rank Can Also Be Represented by a Directed Weighted Graph and Transition Matrix as Shown Below:

This is a transition matrix created based on the graph to the left.

This entry of \( \frac{1}{2} \) represents the weight of the edge arriving at vertex 3 and leaving vertex 4.

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This is a transition matrix created based on the graph to the left.

- Each column represents the weight of the arrows leaving a particular website.
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Thanks for Listening!