Quaternion-Based Analytical Inverse Dynamics for the Human Body

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Inverse Dynamics for the Human Body

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- It's hard to measure these internal dynamics directly, so we need numerical models.

openoregon.pressbooks.pub

Why We Should Care

Prosthetic Design

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Prosthetic Design

Physical Therapy

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Prosthetic Design

Physical Therapy

Human-Inspired Robots

reddit.com/r/VioletEvergarden, static01.nyt.com, engadget.com

Overview

1. Existing methods and models

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2. Theoretical background

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3. Our novel method

Some Terminology

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ICS, SCS: The inertial coordinate system (ICS) and segment coordinate system (SCS) are the global and segment-specific coordinate axes, respectively.

The Traditional Way of Doing Things

Normally, we start o with a diagram like this:

The Traditional Way of Doing Things

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However, there are some shortcomings to this method.

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- This structure suffers from singularities, or gimbal lock

en.wikipedia.org/wiki/Euler_angles

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- Quaternions define a unique transformation and orientation in terms of an axis and an angle

- The corresponding quaternion to such a transformation is

$$q = \cos\left(\frac{\theta}{2}\right) + \sin\left(\frac{\theta}{2}\right)\vec{u}$$

danceswithcode.net/engineeringnotes/quaternions/

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- Quaternions are also very convenient for rotating vectors, as we simply conjugate them
  - Conjugating a vector simply entails multiplying it by the quaternion and its conjugate, in order:qvq

The One Step Inverse Dynamic Method

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- Screws are really just a concatenation of two specific kinds of vectors: a linear one and a related angular one
- In our case, we use the wrench, which is a force and moment vector combined.
- Screw algebra gives a single step method of calculating the wrench at each subsequent joint (Dumas, 2004):

$$\begin{bmatrix} \vec{F}_i \\ \vec{M}_i \end{bmatrix} = \begin{bmatrix} m_i E_{3\times3} & 0_{3\times3} \\ m_i \tilde{c}_i & I_i \end{bmatrix} \begin{bmatrix} \vec{a}_i - \vec{g} \\ \vec{\alpha}_i \end{bmatrix} + \begin{bmatrix} 0_{3\times1} \\ \tilde{\vec{\omega}}_i I_i \vec{\omega}_i \end{bmatrix} + \begin{bmatrix} E_{3\times3} & 0_{3\times3} \\ \tilde{d}_i & E_{3\times3} \end{bmatrix} \begin{bmatrix} \vec{F}_{i-1} \\ \vec{M}_{i-1} \end{bmatrix}$$

The Basic Model: A Diagram and a Brief Explanation

The Basic Model: Equations

We assume for now that the arm is not in motion. Then in this framework, all moments and forces can be obtained from a sequence of matrix products:

Wrist: $$\begin{bmatrix} F_1 \\ M_1 \end{bmatrix} = \begin{bmatrix} -m_0 g \\ \vec{0} \end{bmatrix}$$

Elbow: $$\begin{bmatrix} F_2 \\ M_2 \end{bmatrix} = \begin{bmatrix} m_1 E_{3\times 3} & 0_{3\times 3} \\ m_1 \tilde{c}_1 & I_1 \end{bmatrix} \begin{bmatrix} g \\ 0 \end{bmatrix} + \begin{bmatrix} E_{3\times 3} & 0_{3\times 3} \\ \tilde{d}_1 & E_{3\times 3} \end{bmatrix} \begin{bmatrix} F_1 \\ M_1 \end{bmatrix}$$

Shoulder: $$\begin{bmatrix} F_3 \\ M_3 \end{bmatrix} = \begin{bmatrix} m_2 E_{3\times 3} & 0_{3\times 3} \\ m_2 \tilde{c}_2 & I_2 \end{bmatrix} \begin{bmatrix} g \\ 0 \end{bmatrix} + \begin{bmatrix} E_{3\times 3} & 0_{3\times 3} \\ \tilde{d}_2 & E_{3\times 3} \end{bmatrix} \begin{bmatrix} F_2 \\ M_2 \end{bmatrix}$$

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The middle wrench is gone, as are both acceleration vectors:

Adding Musculature

Before adding additional segments, we introduce a framework for incorporating muscles into the model.

Each muscle is treated as a tension between two xed endpoints.

Changes to the Algorithm

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3. Recalculate the wrenches with a modi ed equation:

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- Our current algorithm does not have a simple way to let segments converge
- There are exponentially more possible ways to hold an object as segments increase in number

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- Our indices are now simply a labelling serving as a way to distinguish points
- We create a hierarchy based on the number of edges between the shoulder and each point.
- Calculations now iterate along each hierarchy number

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- Each additional segment we add to the model creates more degrees of freedom
- The human body usually uses the same orientation to hold an object
- We create a rudimentary way of predicting how the arm will naturally position itself

Simu Liu Stock Photo

Orienting the Segments

The gist of it is that we nd whatever orientation minimizes bending at the wrist:

Orienting the Segments

Recap

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- Screw algebra for efficiency
- Muscle integration
- Implementation of the hand

Acknowledgements

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