

# **The Power of Many: A *Physarum* Swarm Steiner Tree Algorithm**

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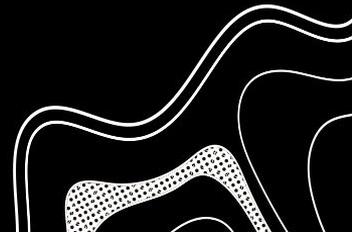
Mentored by Laura Schaposnik, University of Illinois at Chicago

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# Large Scale Hard Problems

- NP, NP hard, NP complete problems
  - Cannot be solved in deterministic polynomial time
- Example: Traveling Salesman Problem
  - Given list of cities & distances, find shortest route to visit all of them & return to start
  - Many applications - UPS package delivery, school bus routes, drilling holes in circuit board, subproblem of DNA sequencing
- Difficult because large solution space to search



The background features a complex, abstract pattern of white lines and dots on a black background. The lines are irregular and wavy, creating a sense of movement and depth. Some areas are filled with a dense pattern of small white dots, while others are solid black or white. The overall effect is reminiscent of a microscopic view of a biological structure or a topographical map.

## NEW APPROACH

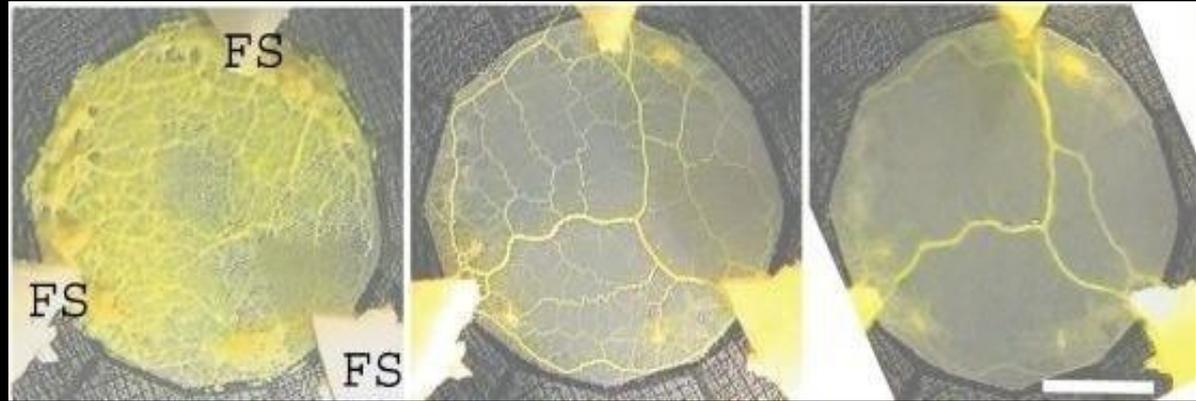
Use swarms of *Physarum* to locally solve problem before fusing and solving entire problem

**PHYSARUM**  
**POLYCEPHALUM**

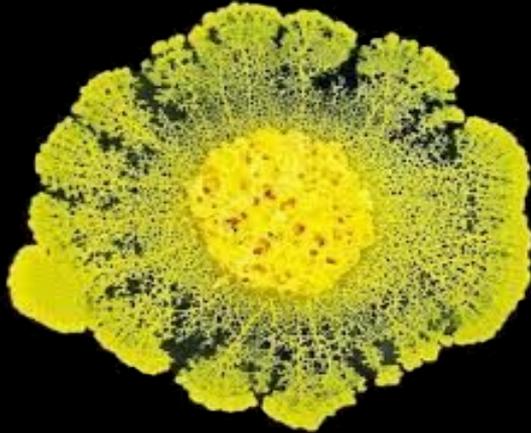


# PHYSARUM POLYCEPHALUM

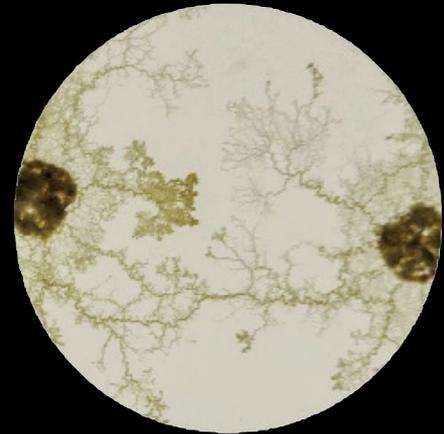
Steiner trees



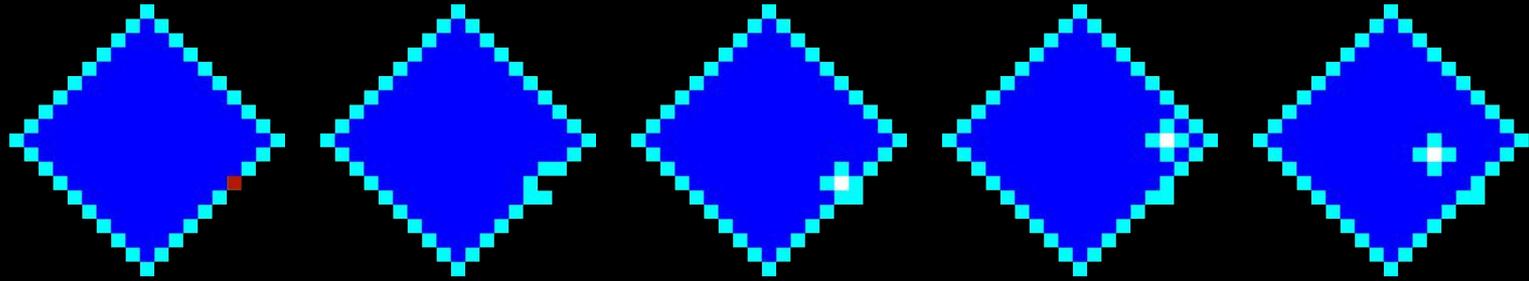
Unicellular



Fusion

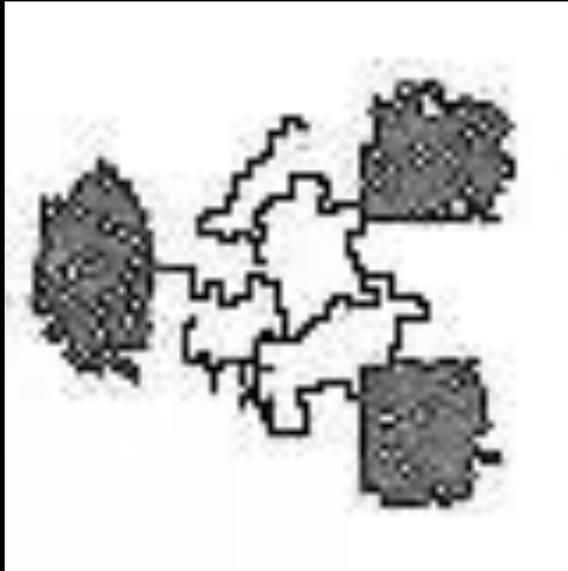


# CELL Model

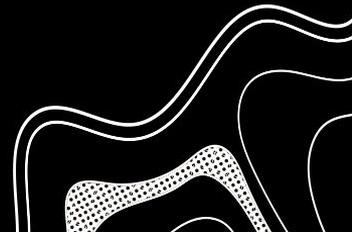


- Model on grid
- 2 = cytoskeleton, 1 = cytoplasm, 0 = outside
- At every iteration:
  - Bubble (piece of outside, state 0) introduced into the organism at stimulus point
  - Bubble slowly moves throughout organism
- By introducing bubbles many time, CELL can move and reshape

# Stimulus Point

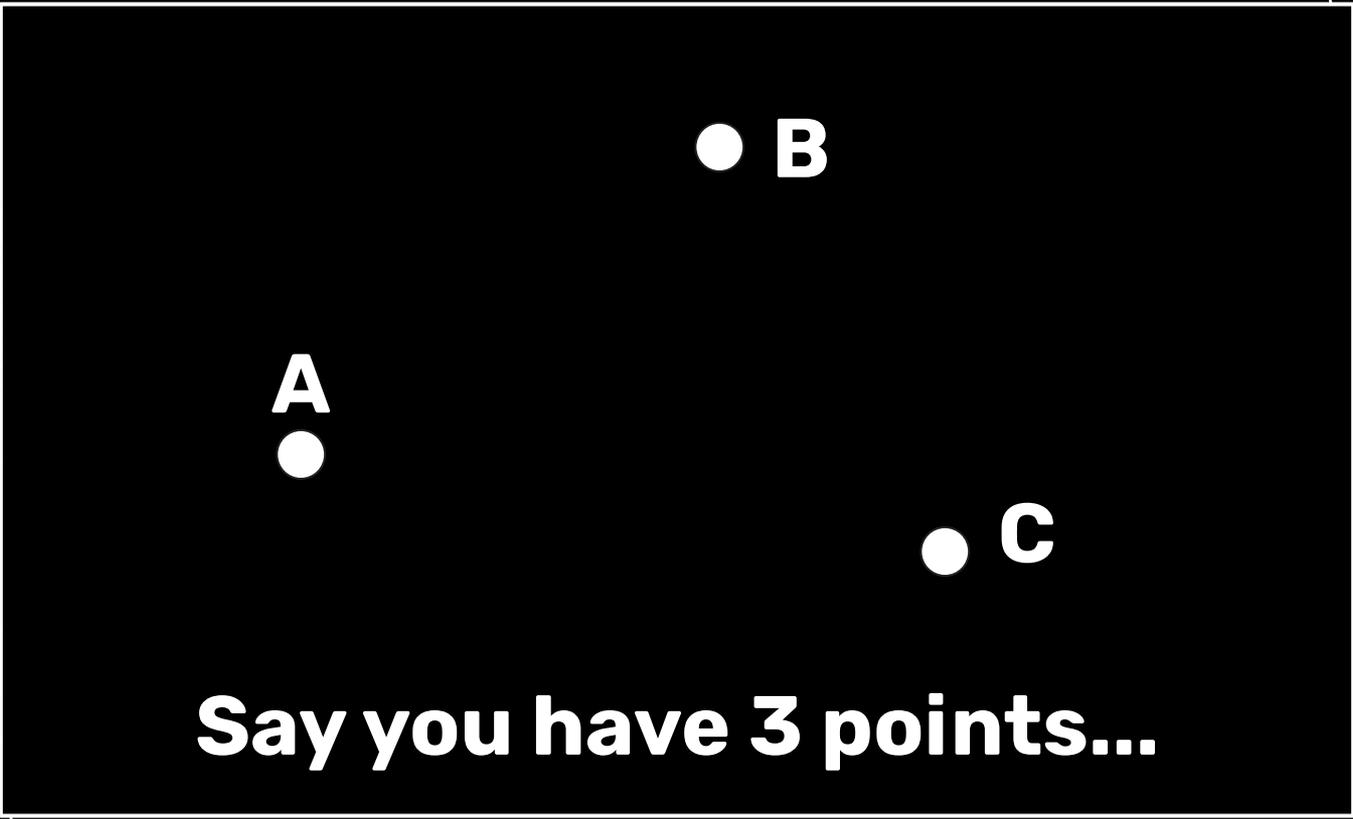


- Stimulus point - 2, choose point bubble is introduced
- Randomly chosen - amoebic motion
- Chosen from active zones - tree
  - Always choose stimulus point from certain area
  - Moves cytoplasm into the area



# Model of Multiple CELLS





**A**

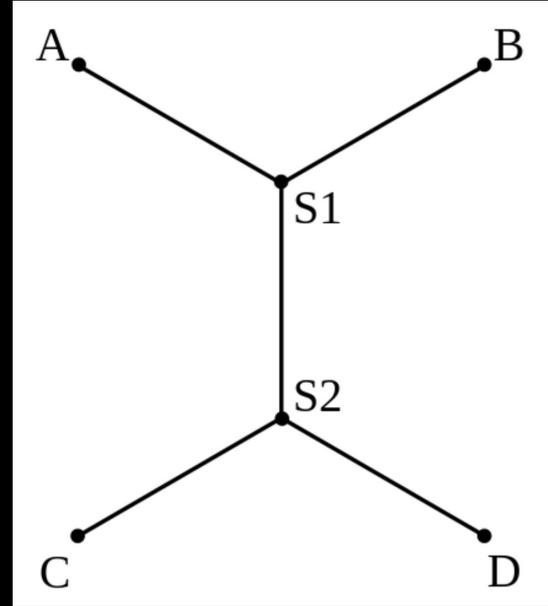
**B**

**C**

**Say you have 3 points...**

# Steiner Trees

- Set of points (terminals), find minimum length tree connecting them. Can contain other points (steiner points)
- NP hard
- Researched since 19th century, current leading software still takes up to 7 days for large graphs
- Many applications: VLSI design, transportation and cabling networks

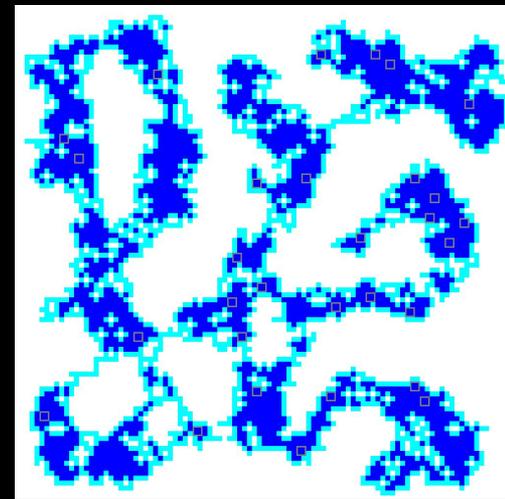
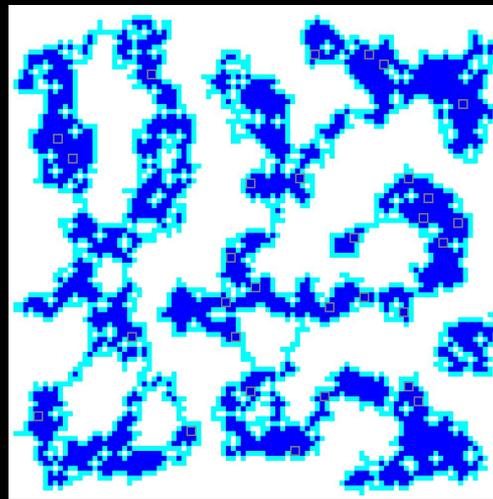
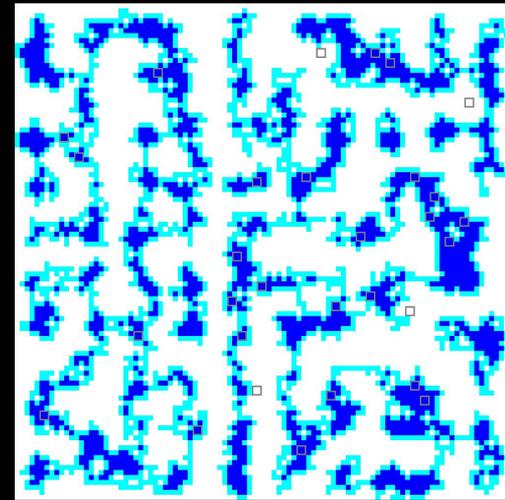
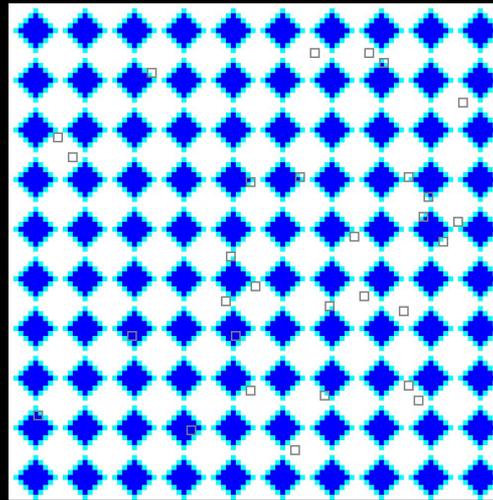


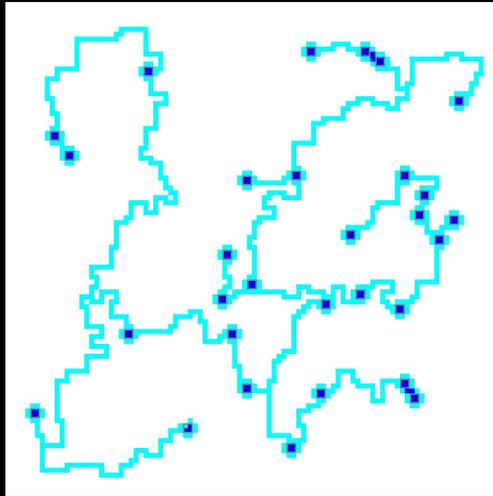
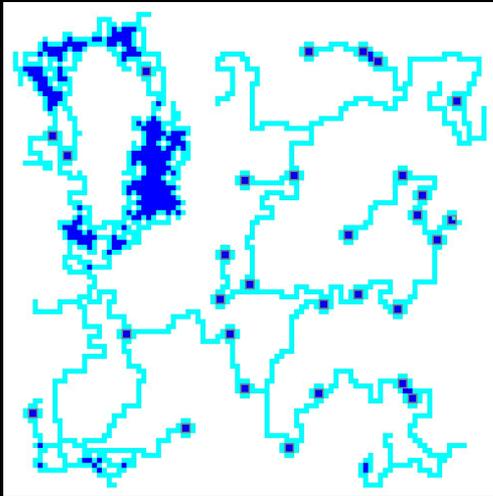
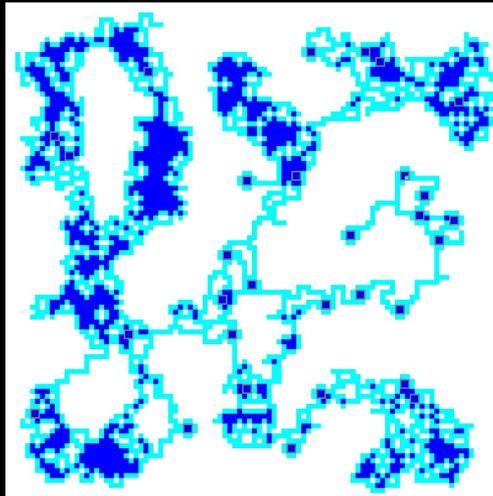
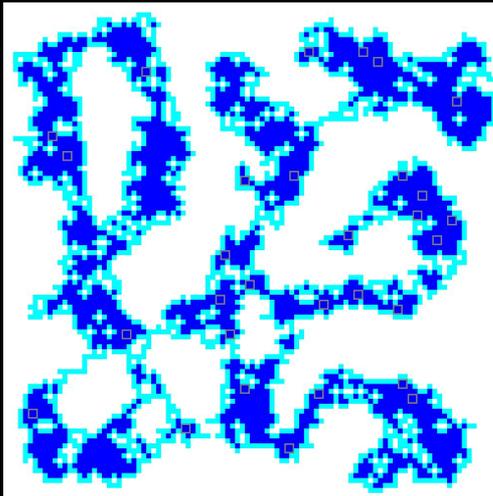
# Physarum Steiner Algorithm



# Foraging

- One cell finds all points
- Two options: choose stimulus point from active zone or randomly
- Choice is controlled by probability formula





## Shrinking

- Find minimal tree
- Remove cytoplasm from inside points when no stimulus point

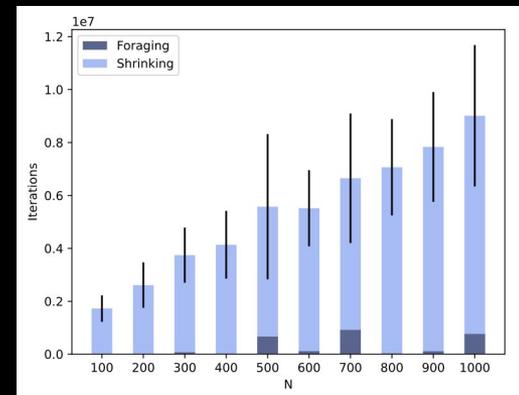
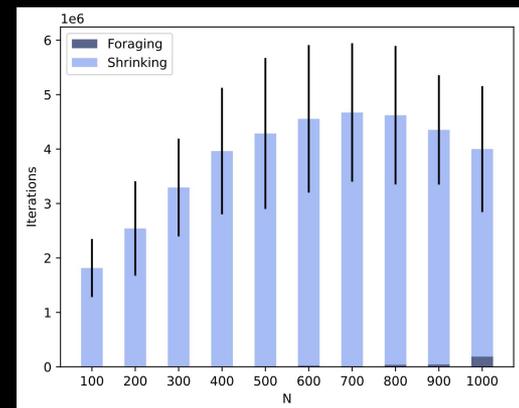
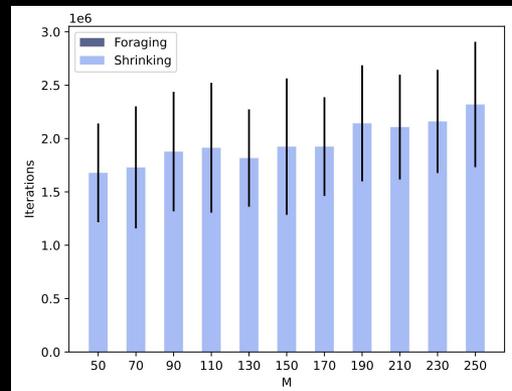
# Time Complexity

Iterations, time per iteration is not necessarily constant

With respect to  $N$ : less than linear

With respect to  $M$ : linear

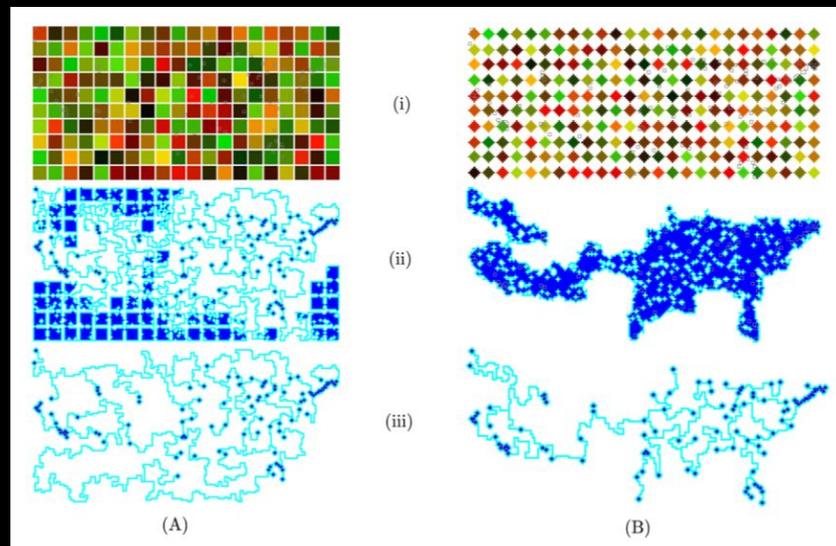
$M$  varies with  $N$ : linear



# CELL SHAPE

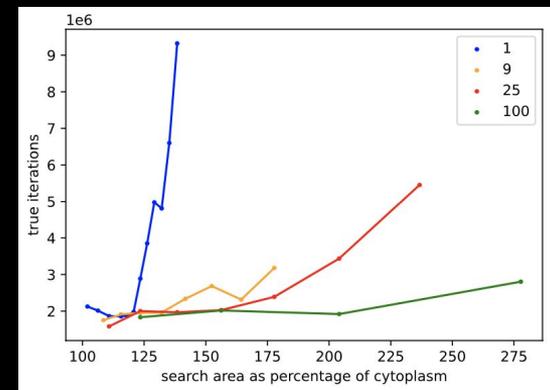
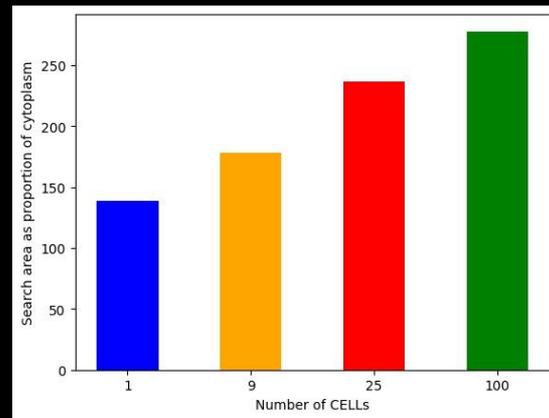
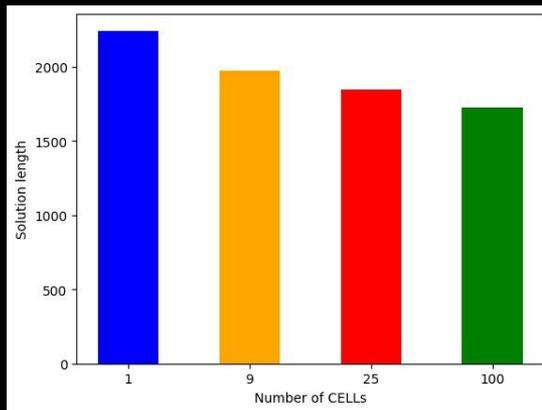
Squares result in faster runtimes, better fit for square graphs

Diamonds use less cytoplasm, result in better solutions



# Multiple CELLs

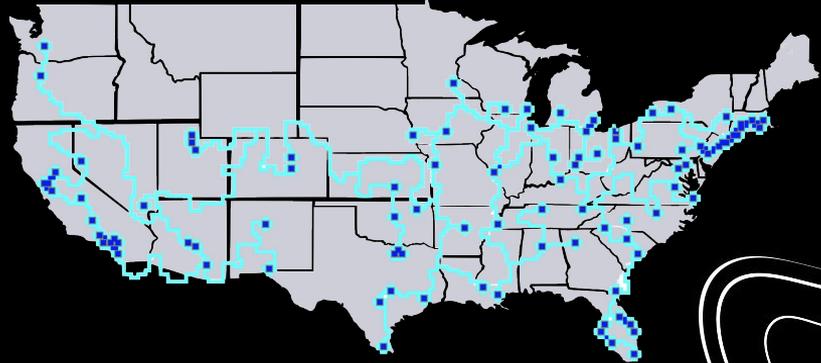
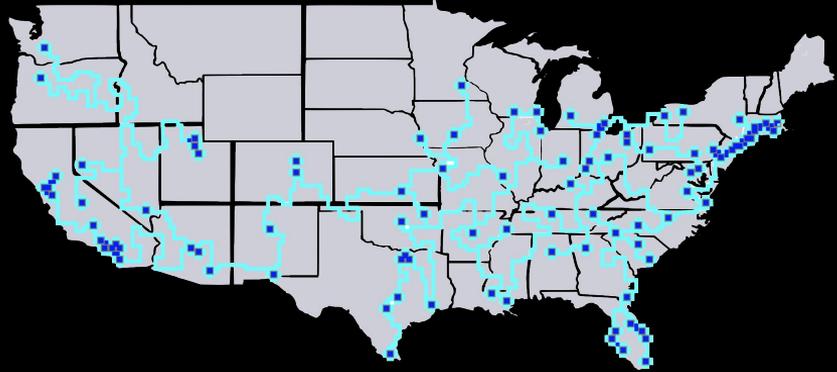
Using more cells allows us to explore bigger search areas, find shorter solutions, and solve problems faster



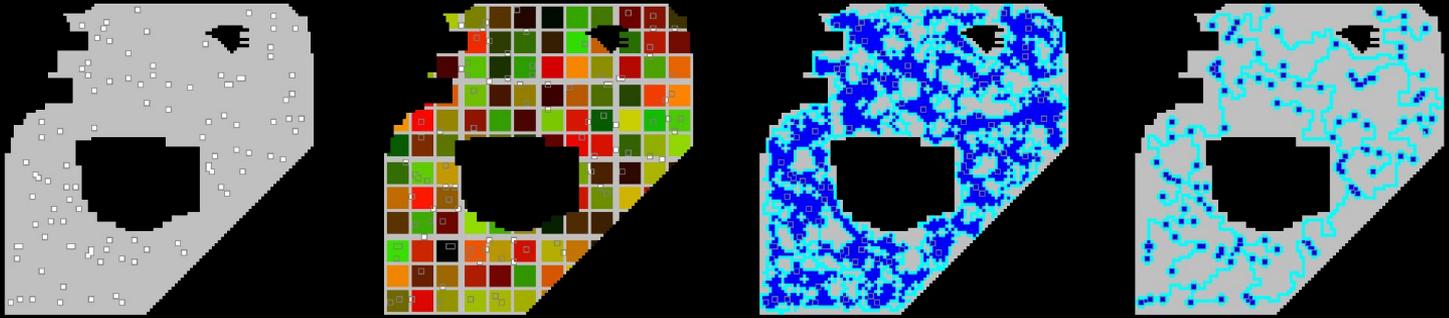
# Road Networks

Create network between  
100 largest cities, lower 48

Gradually solves Steiner  
Tree, redundancy

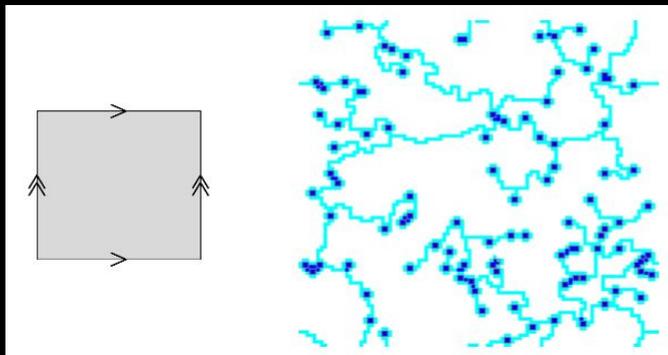


# Obstacle Avoidance

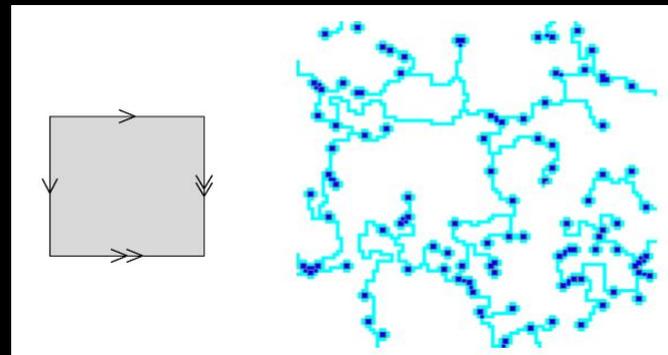


Lay cabling in residential neighborhood, avoiding obstacles such as a lake and park

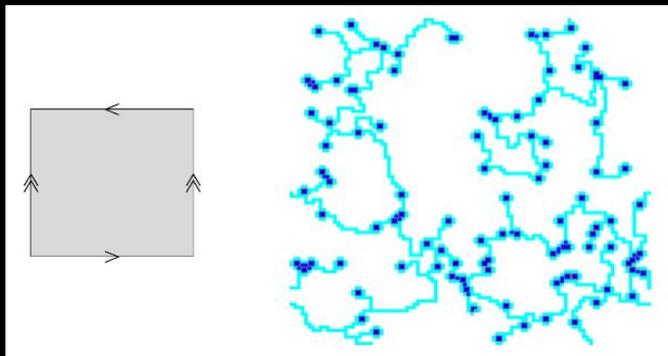
# Topological Surfaces



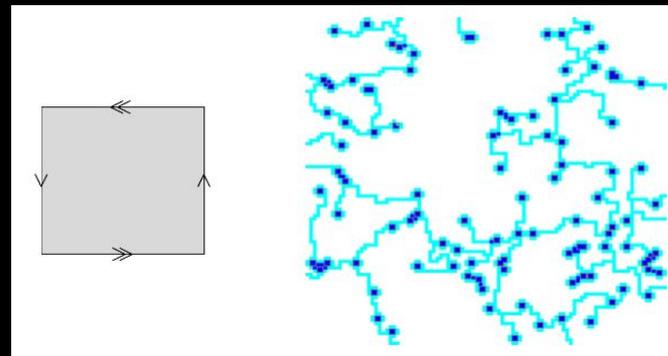
Torus



Sphere



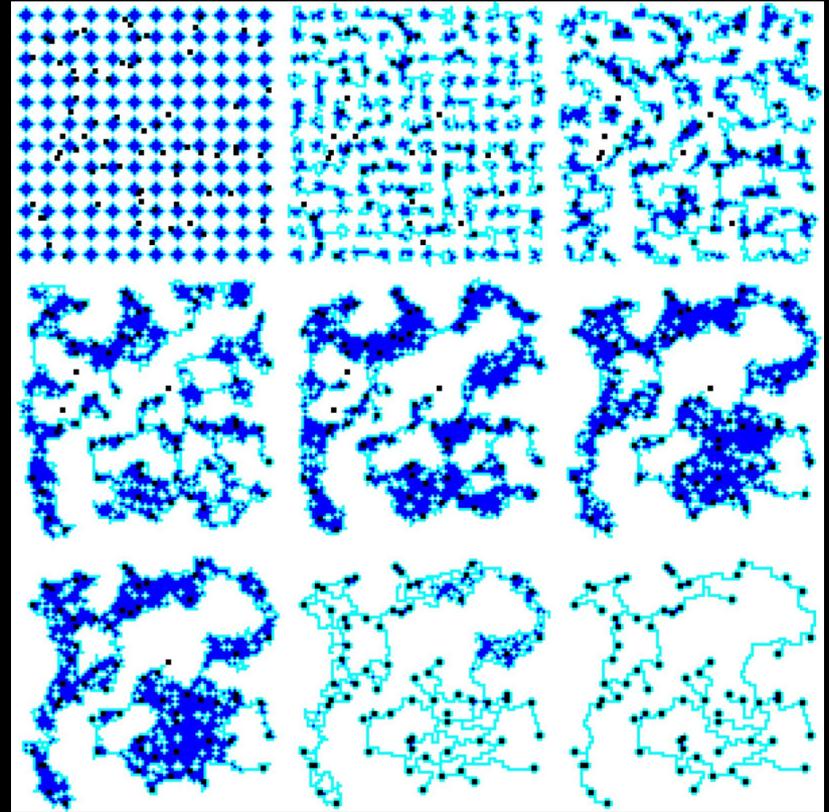
Klein Bottle



$\mathbb{RP}^2$

# Conclusion

- Noteworthy abilities: obstacle avoidance, redundancy
- Physarum Swarms can be applied to other difficult computing problems



# Acknowledgements

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- Family

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