Thermocapillary Modulation of Fluidic Lenses in Microgravity

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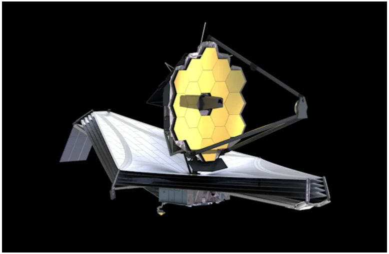
Motivation

The James Webb telescope is launched into space, and then is supposed to unfold.



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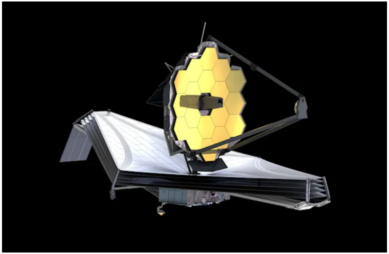

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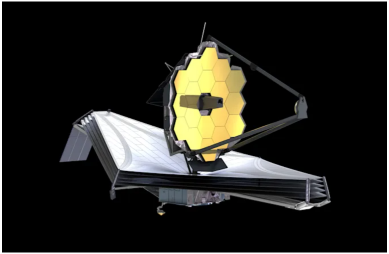

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What if we can form the lens in space?

Using Liquids

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They are also more cost-efficient.

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To make ℓ large, we can either make 0 or g 0.

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First Method

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We just have to minimize the interfacial energy, i.e. the product of the surface tension and surface area.

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$$G(r;\ ) = \quad _0\ f(r;\ ) \left(\sqrt{1 + h_r^2 + \frac{1}{r^2} h^2} + \quad h \right) r;$$

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In order to minimize this, we can use the Euler-Lagrange Equation:

$$\frac{\partial G}{\partial h} \quad \frac{d}{dr}\frac{\partial G}{\partial h_r} \quad \frac{d}{d}\frac{\partial G}{\partial h} = 0:$$

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We introduce $\varepsilon = \left(\frac{h_0}{R_0}\right)^2$. Here, h_0 is the thickness of the lens.

In our situation, ε is very small. This means any term with ε in it is negligible. If we ignore such terms, our system is greatly simplified!

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Surface Equation (Das & Frumkin)

We have

$$PR^2 \quad (R^2 F_R H_R + F \; H \;) \quad F \; (H_{RR} R^2 + R H_R + H \quad) = 0;$$

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If we let $x = R^{p} \overline{|P|}$, we get

$$x^2 \quad (x^2 F_x H_x + F \quad H \quad) \quad F \quad (x^2 H_{xx} + x H_x + H \quad) = 0;$$

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We choose $F(x) = 1 \quad x^2$.

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The solution to the system is

$$H(x) = \; C_2 + \; C_1 \log x \qquad \frac{(1 + \; 2 \;\; C_1) \log(1 \qquad x^2)}{4}:$$

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For di erent functions F, we would get di erent solutions for H. We chose this F because it allows us to solve for H analytically. For more complex functions F, such as functions that are dependent, we could construct more complex lenses, but we may not be able to solve for them analytica

Graphs

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These ows can a ect the shape of the lens in thin lms. Since the temperature gradient remains the same, the ows will be constant.

Another Method

We can use thin lms as a corrective element for normal, spherical thick lenses.

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This is still a work in progress.

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my parents and my brother

References

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