Thermocapillary Modulation of Fluidic Lenses in Microgravity

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October 16-17, 2021
MIT PRIMES Conference
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What if we can form the lens in space?
Using Liquids

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Capillary Length

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**Definition (Capillary Length)**

The capillary length of a liquid is

$$\ell = \sqrt{\frac{\gamma \Delta \rho \cdot g}{}}$$

where $\gamma$ is the surface tension, $\Delta \rho$ is the difference in density between the liquid and the ambient environment, and $g$ is the acceleration due to gravity.
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To make \( \ell \) large, we can either make \( \Delta \rho \approx 0 \) or \( g \approx 0 \).
The Setup

A liquid is injected into a cylindrical bounding frame, with radius $R_0$ and height $d$. We want to find the function $h(r, \theta)$ that represents the interface of the liquid with the outside.

With constant surface tension, the shape of the lens is spherical. We want to slightly deform this shape to correct for spherical aberrations.
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First Method

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Let the surface tension at \((r, \theta)\) be \(\gamma_0 \cdot f(r, \theta)\) for some function \(f\).
Free Energy

Let the surface tension at \((r, \theta)\) be \(\gamma_0 \cdot f(r, \theta)\) for some function \(f\). Let

\[
G(r, \theta) = \left(\gamma_0 \cdot f(r, \theta) \sqrt{1 + h_r^2 + \frac{1}{r^2} h_\theta^2 + \lambda h}\right) r,
\]

\(h_r = \partial_r h\), and \(h_\theta = \partial_\theta h\).
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\Pi = \int_0^{2\pi} \int_0^{R_0} G(r, \theta) \, dr \, d\theta.
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In order to minimize this, we can use the Euler-Lagrange Equation:

\[
\frac{\partial G}{\partial h} - \frac{d}{dr} \frac{\partial G}{\partial h_r} - \frac{d}{d\theta} \frac{\partial G}{\partial h_\theta} = 0.
\]
Introducing $\varepsilon$

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We introduce $\varepsilon = \left( \frac{h_0}{R_0} \right)^2$. Here, $h_0$ is the thickness of the lens.

In our situation, $\varepsilon$ is very small. This means any term with $\varepsilon$ in it is negligible. If we ignore such terms, our system is greatly simplified!
Our Results

The final equation we get is the following.

\[
PR^2 - (R^2F + RH + F\Theta H) - F \cdot (H_\text{RR}R^2 + RH + H_\Theta \Theta) = 0,
\]

where \(P\) is a constant of our choice.

If we let \(x = R\sqrt{|P|}\), we get

\[
\pm x^2 - (x^2F + \Theta H - F \cdot (x^2H + \Theta)) = 0,
\]

where the sign of the first term is determined by the sign of \(P\).

We choose \(F(x) = 1 - \beta x^2\).
Our Results

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**Surface Equation (Das & Frumkin)**

We have

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where \( P \) is a constant of our choice.

If we let \( x = R \sqrt{|P|} \), we get

\[ \pm x^2 - (x^2 F_x H_x + F_\Theta H_\Theta) - F \cdot (x^2 H_{xx} + x H_x + H_{\Theta\Theta}) = 0, \]

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We choose \( F(x) = 1 - \beta x^2 \).
The solution to the system is

\[ H(x) = C_2 + C_1 \log x \mp \frac{(1 + 2\beta C_1) \log(1 - \beta x^2)}{4\beta}. \]
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For different functions \( F \), we would get different solutions for \( H \). We chose this \( F \) because it allows us to solve for \( H \) analytically. For more complex functions \( F \), such as functions that are \( \Theta \) dependent, we could construct more complex lenses, but we may not be able to solve for them analytically.
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These flows can affect the shape of the lens in thin films. Since the temperature gradient remains the same, the flows will be constant.
Another Method

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This is still a work in progress.
Acknowledgements

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- my parents and my brother
References


