Mathematical and Algorithmic Models of Refugee Crises

Vincent Huang
Mentor: James Unwin, University of Illinois

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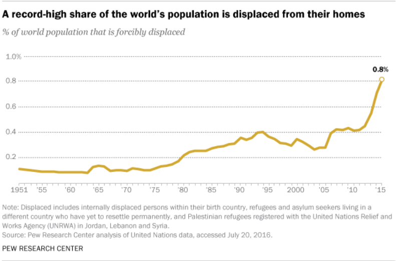
Importance

What and Why:

65.6 Million Refugees in the Past Year

- Asylum Registration
- Legal Enforcement
- Humanitarian Aid
    - Limited Resources
- Contingency Planning



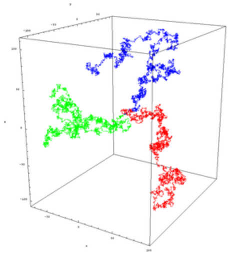
Displaced Population Over Time (Source: Pew Research Center)

Introduction to Agent-Based Models (ABMs)

- Popular recently due to versatility and increased access to computing power

Two Primary Components:

- System: the "map" or "space" to be modeled, usually some sort of graph
- Agents: react to each other and to the system via simple algorithmic rules

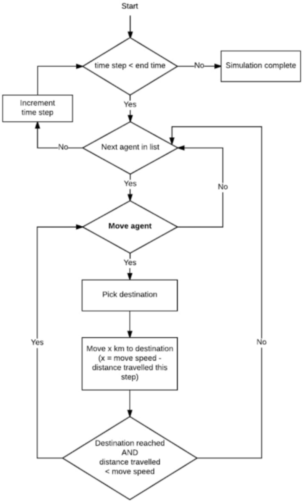


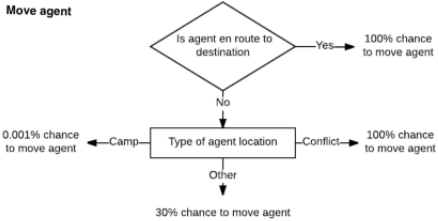
Random walks (Source: Wikipedia)

Classical examples: Random walks, Epidemic spread, Oil prices

Suleimenova-Bell-Groen Approach

We examined a specific agent-based model called Flee, developed by Suleimenova-Bell-Groen. This approach has agents move to adjacent cities with probability proportional to $(\text{distance between cities})^{-1}$.

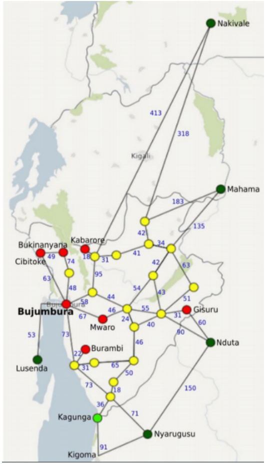




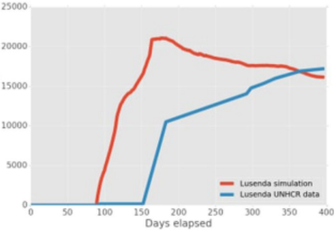
Suleimenova, Bell, & Groen. Scientific reports 7, 1 (2017): 13377.

Results of Flee

Flee accurately predicts 75% of the end distribution of refugees. Demonstrated on three conflicts: Burundi, the Central African Republic, and Mali. Initially we have limited our focus to Burundi.



Map of Burundi used in Flee

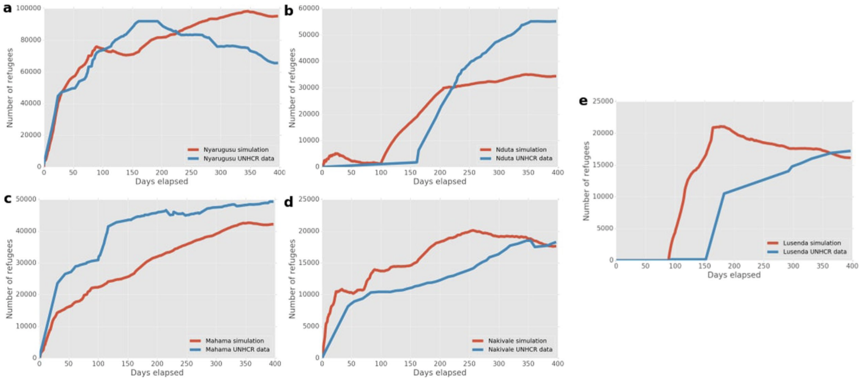


Flee vs. real data in Lusenda

- Copies basic features and end distribution although with significant discrepancy in the middle.

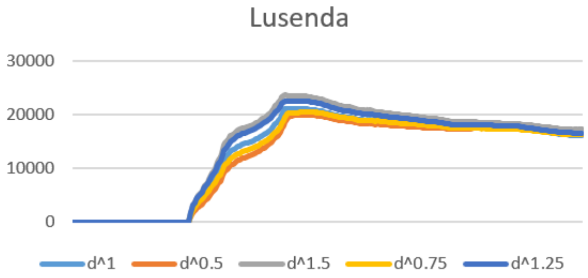
Results of Flee, Continued

Global picture of system is found by looking at all camps simultaneously:



Variations on Flee

We replaced the probability weightings of (distance between cities)$^{-1}$ with (distance between cities)$^{-n}$ for $n = \frac{1}{2}, \frac{3}{4}, \frac{5}{4}, \frac{3}{2}$.

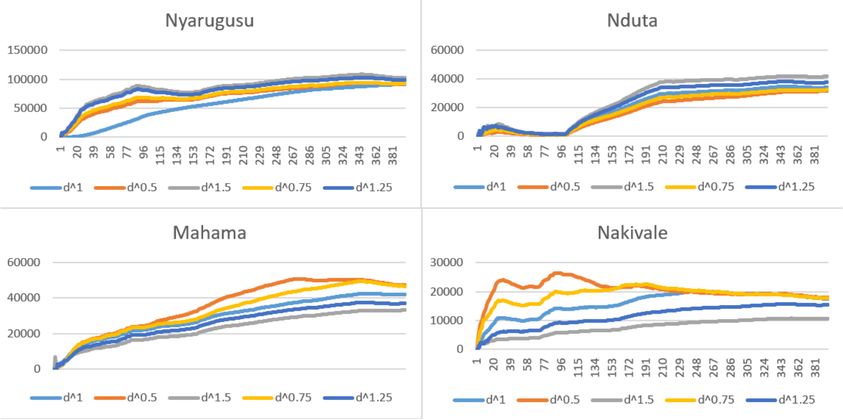


Flee vs. real data in Lusenda

- Some variation, but not that much
- No clear patterns (e.g. not "monotonic")
- Suggests there are ways to improve upon Flee

Variations on Flee, Continued

- Same trends as before
- Output is fairly resistant to changes in Flee algorithm



New Approach: Stochastic Matrices

We can use stochastic matrices to model systems assuming:

- A series of fixed states $X_1, X_2, ..., X_n$
- Transition probabilities $P_{j,i}$ of transitioning from states $X_i \rightarrow X_j$
- Initial state X_i with probability Q_i

Represent the system's initial state and its transition probabilities by

$$B_0 = \begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_n \end{bmatrix}, \quad \text{and} \quad A = \begin{bmatrix} P_{1,1} & P_{1,2} & \dots & P_{1,n} \\ P_{2,1} & P_{2,2} & \dots & P_{2,n} \\ \vdots & \vdots & \dots & \vdots \\ P_{n,1} & P_{n,2} & \dots & P_{n,n} \end{bmatrix}.$$

The recurrence $B_k = AB_{k-1}$ describes the system at step k.

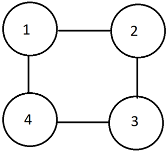
Surprisingly, applications of stochastic matrices to the problem of migration have not been attempted before.

Application to Migration

A system similar to Flee can be implemented via stochastic matrices:

- Compute transition probabilities $P_{j,i}$ of moving from city i to city j similar to Flee. Record probabilities in the stochastic matrix A.
- Let B_0 be an $n \times 1$ matrix representing the distribution of refugees at time 0.
- Compute the distribution $B_k = AB_{k-1}$ of refugees at time k;
- Update refugee populations and transition probabilities accordingly.

Example

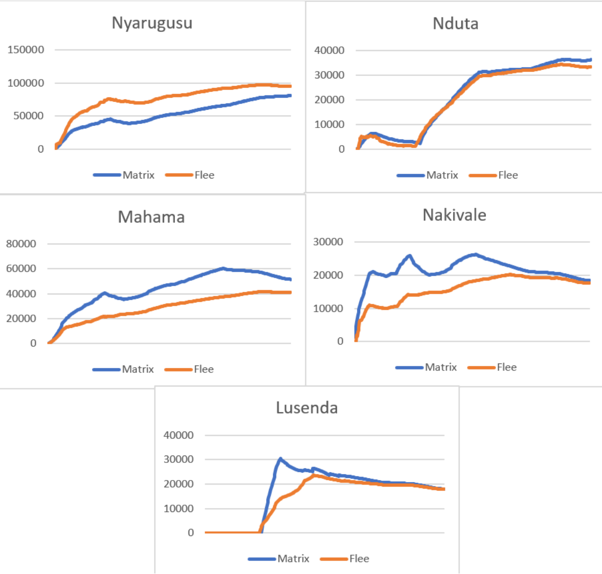


Corresponding stochastic matrix:

$$\begin{bmatrix} 0 & 0.5 & 0 & 0.5 \\ 0.5 & 0 & 0.5 & 0 \\ 0 & 0.5 & 0 & 0.5 \\ 0.5 & 0 & 0.5 & 0 \end{bmatrix}$$

Stochastic Matrix Results

- Copies most of the features from Flee
- Moderate differences due to changes in implementation



Summary and Future Directions

We examined models based on ABM and stochastic matrices. Stochastic matrices should yield further fruitful results because:

- Significantly shorter code, fewer computations, and faster runtimes make debugging and variation easier.
- Matrix model is deterministic and relatively simple to analyze.
- There is a large body of mathematical literature on stochastic matrices which may be applied towards migration problems.

We will use stochastic matrices to progress towards the goals below:

- Improving matching between simulations and data;
- Mathematically analyzing our model - for example: rate of convergence and end behavior;
- Considering the optimization of a given graph or network.

Acknowledgments

Key References:

- Suleimenova, Bell, & Groen. “A generalized simulation development approach for predicting refugee destinations.” Scientific reports 7, 1 (2017): 13377.
- Gulden, Harrison, & Crooks, "Modeling Cities and Displacement through an Agent-based Spatial Interaction Model", Computational Social Sciences, 2011.
- Gagniuc, Paul A. (2017). “Markov Chains: From Theory to Implementation and Experimentation.” USA, NJ: John Wiley & Sons.

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Thank you!