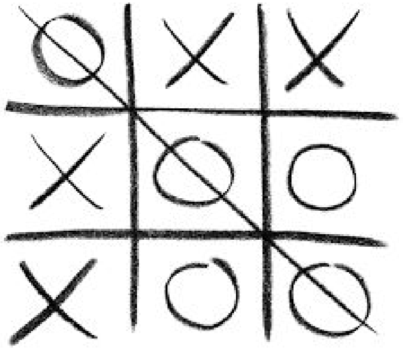
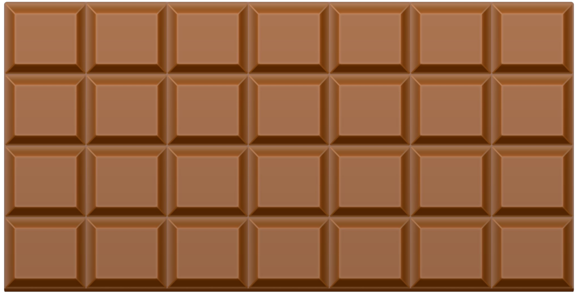
MIT PRIMES STEP 2016-2017

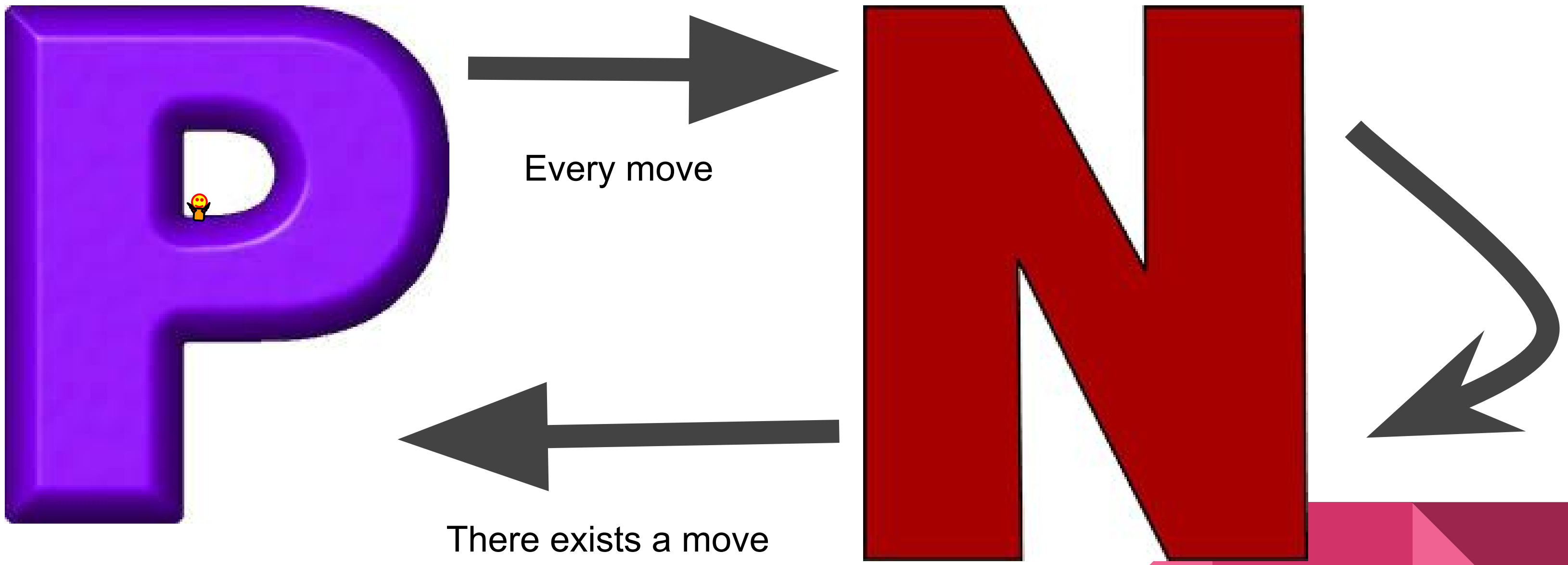
Impartial combinatorial games

Impartial games





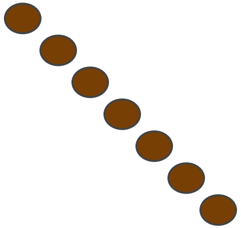
P and N positions.



The Chocolate Stones Game
Suppose N = 2 and P = 7

Player 1

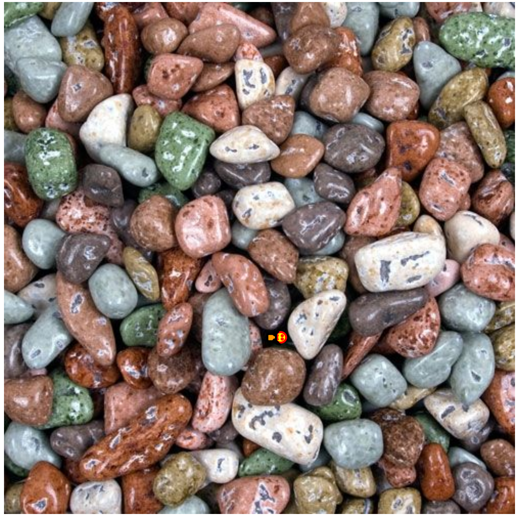
Player 2



Step 1: Take 1 stone since P mod N is 1.
Step 2: Player 2 is forced to take 2.
Step 3: Player 1 is forced to take 2.
Step 4: Player 2 is forced to take 2.
Step 5: Player 2 wins as Player 1 has no move.

No-Strategy Games

- A no strategy game is a game in which the same person will win no matter what they do.
- Chocolate Stones is an example of a No Strategy Game

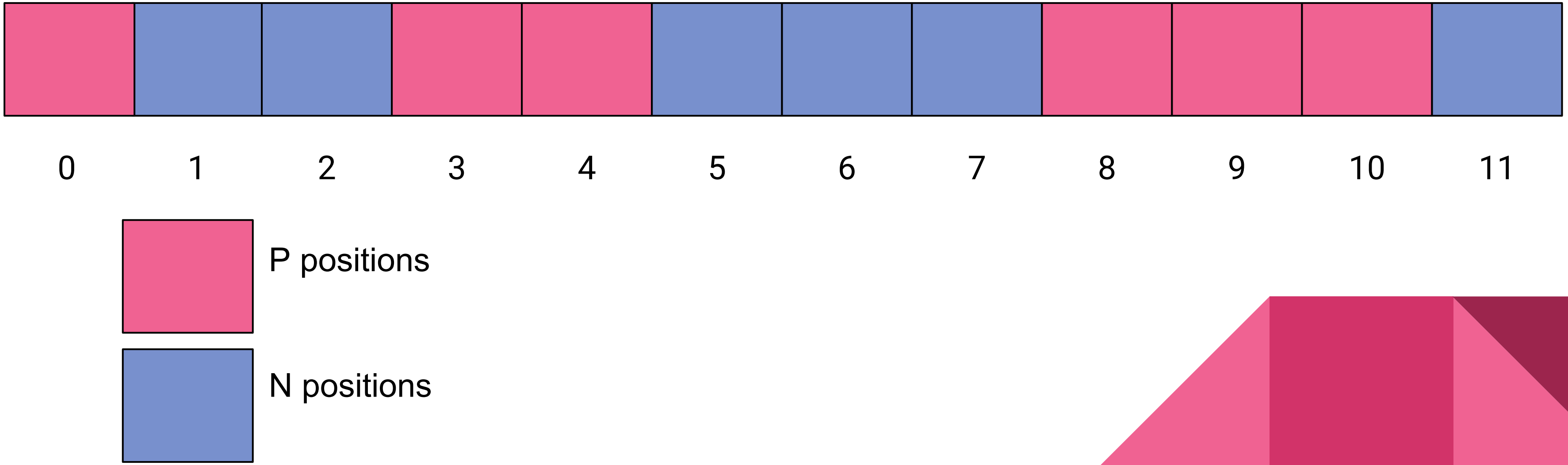


Demon Money (Square Root) Game

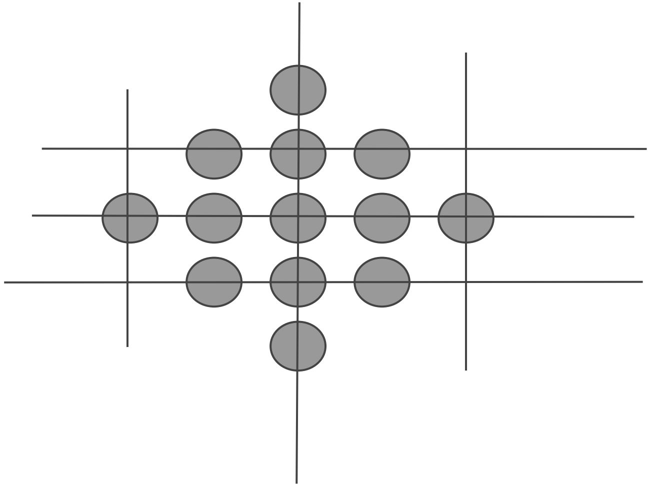
You and your friend live in a city ruled by evil demons. Every day, the demons require you to pay the square root of their money in taxes. However, because the demons do not like decimals, they require the taxes to be rounded to a whole number. The person filing the taxes may choose to round the amount paid up or down. The person who pays the demons $0 gets executed by the tyrannous demon king.

The P positions:

We have proved that, for integers k>1, P positions are in the interval $[k^2-1, k^2+k-2]$. This can be shown by segments along a line. In an optimal game, each move goes from one segment to the next.



Diamonds Game:



1

2

How to win:

- If the points form two parallel lines, take a line perpendicular to them.
- If the other player takes one of the axes, take the other.
- If the other player takes a horizontal line other than the x axis, take that line reflected across the x axis.
- If the other player takes a vertical line other than the y axis, take that line reflected across the y axis.
- If the points form a single line, take it. (endgame)

But who wins for other shapes?

$N>0$
$M>0$
$(N+M) \bmod 2=0$

Sum from the product game

-Choose a number n.

-P1 Chooses positive integers a and b, such that ab = n, and (n - a - b) > 0. The new number is n = (n-a-b)

-P2 chooses a new a and b for n, and subtracts their sum from n.

-This continues until a player does not have a move. That player loses.

-The P positions form the following sequence: 1, 2, 3, 4, 5, 7, 11, 13, 16, 17, 19, 22, 23, 25, 27, 29, 31,32… . This sequence has been added to OEIS.

No-Factor Game

1 2 3 4 5 6 7 8

Who wins?

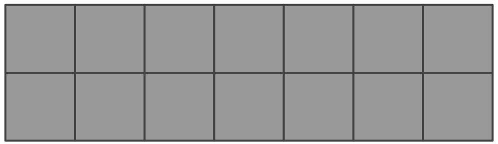
1 2 3 4 5 6

Remove a Square

- Suppose you take a shape made up of squares on a rectangular grid
- each player may remove one square on the grid of any size. The player who removes all of the shape wins. We mostly studied this game for 2 by n rectangles.

Example, 2 x 7:

1

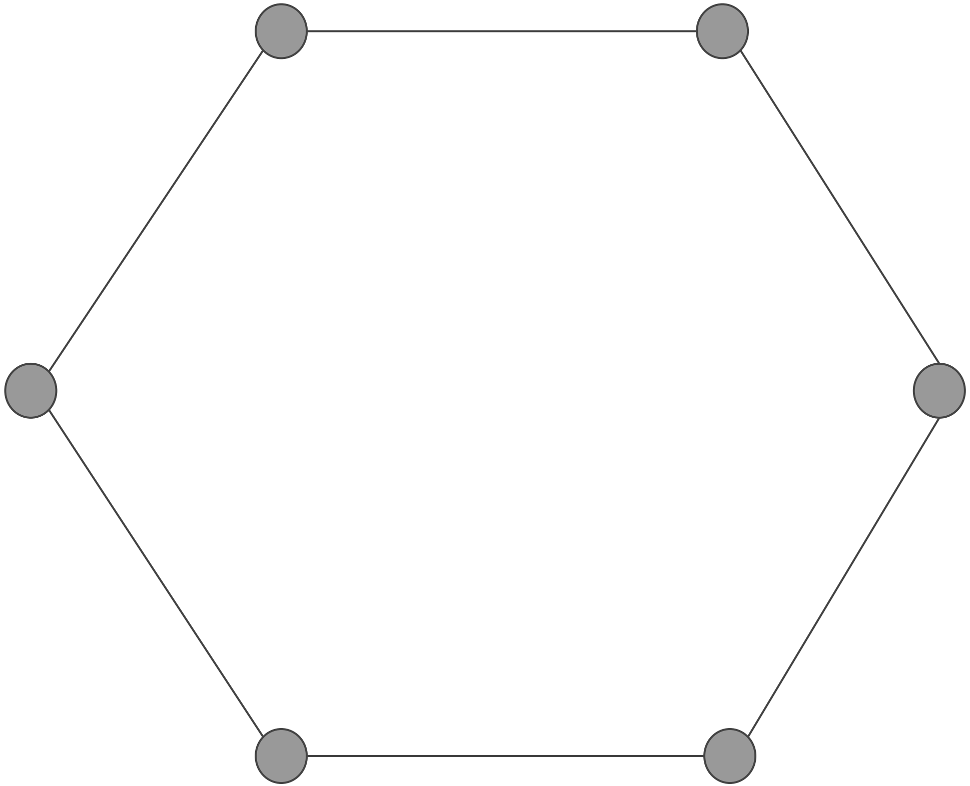


2

Remove an Edge

- Create a graph G with no loops, no double connections.
- Each turn a player must choose two vertices and remove all edges which contain those two vertices.
- The player without a move loses.

Remove an Edge example



Step 1: Take any edge.

Step 2: With the new path graph of length 4

The Second player made a mistake, as taking the middle edge would have won him the game

The First player takes advantage of his mistakes and wins

Game End: Player 1 has won the game

Acknowledgements

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- Our Parents
- Cats :)

The End