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is Guilty?

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PRIMES CONFERENCE 5/21/16

THE TYPES

Absolute truth-teller: A person who always tell the truth.

Partial truth-teller: A person who always tells the truth, except when they are guilty and they supply information about their guilt; then they lie.

Absolute liar: A person who always lies.

Responsible liar: A person who always lies, except when they are guilty and they supply information about their guilt; then they tell the truth.

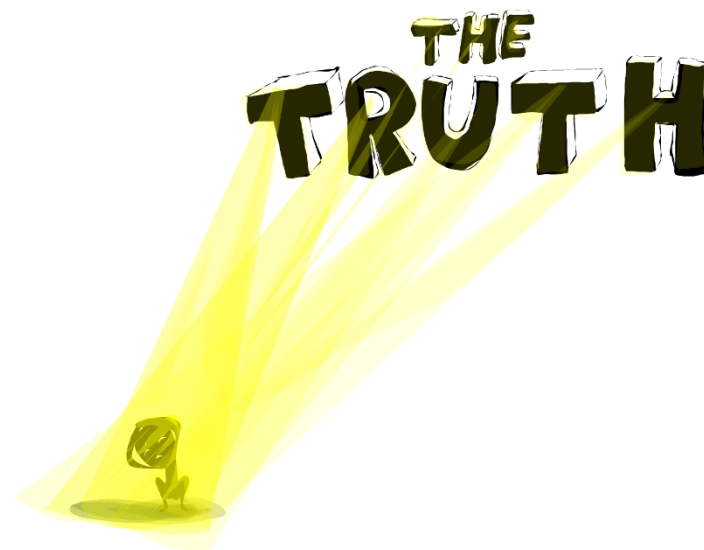
Example question-Are you guilty?

Type	Guilty/Not	Answer
Absolute truth-teller	Guilty	Yes.
	Not Guilty	No.
Partial truth-teller	Guilty	No.
	Not Guilty	No.
Absolute liar	Guilty	No.
	Not Guilty	Yes.
Responsible liar	Guilty	Yes.
	Not Guilty	Yes.

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Example Question- Is “Person A” guilty?

Type	Person A is Guilty/Not/Don't know	Answer
Absolute truth-teller	Guilty	Yes.
	Not	No.
	Don't know	Don't know.
Partial truth-teller	Guilty	Yes.
	Not	No.
	Don't know	Don't know.
Absolute liar	Guilty	No.
	Not	Yes.
	Don't know	Any answer except “Don't know”
Responsible liar	Guilty	No.
	Not	Yes.
	Don't know	Any answer except “Don't know”



Truth-Tellers island

Starring: Ashwin and Jacob

First Ideas

Lemma. On the Truth-Tellers Island if someone says they committed the crime, then they actually committed the crime. In addition, statements about other people are always true.

Lemma. On the Truth-Tellers Island if the crime was committed by at least two people together who didn't hide their identities from each other, the detective can always solve the crime.

Proof. The detective asks each person about the guilt of all the other suspects. Since the Truth-Teller islanders do not lie about other people, each criminal will be identified by his partners.

NEIL'S PUZZLE

Detective Khovanova knows from her previous interrogations that the stealing of the cuff-links was committed by exactly one person out of four prime suspects. The suspects have never met or heard about each other. In response to the detective's question, the suspects answer as truthfully as possible to the extent of their knowledge. The same question which solved the crime was asked to everyone. Which one?



The mayor's favorite cuff-links which are #42 in her office

NEIL'S SOLUTION

Any question such as “Did I do it?” from the detective work.



The mayor's favorite cuff-links which are #42 in her office

BIG THEOREM: On the truth-tellers island it is possible to solve any crime. The steps:

1. Detective asks everyone about other people
2. All the people who didn't hide their identity are found
3. Detective asks a question such as "Did someone else commit the crime?"
4. All criminals are found





THIS GREEN
APPLE IS RED

Liars island

Starring: Jonathan and Leon

Our ideas

We can translate our truth-teller island lemmas to the liars island. However, the questions that we ask have to be remade into yes/no questions. For example:

Lemma. On the Liars Island if someone says they are not guilty, then they are guilty. In addition, statements about other people are false.

Lemma. On the Liars Island if for every criminal there is at least one other person who knows that this criminal committed the crime, then the crime can be solved.

JONATHAN'S PUZZLE

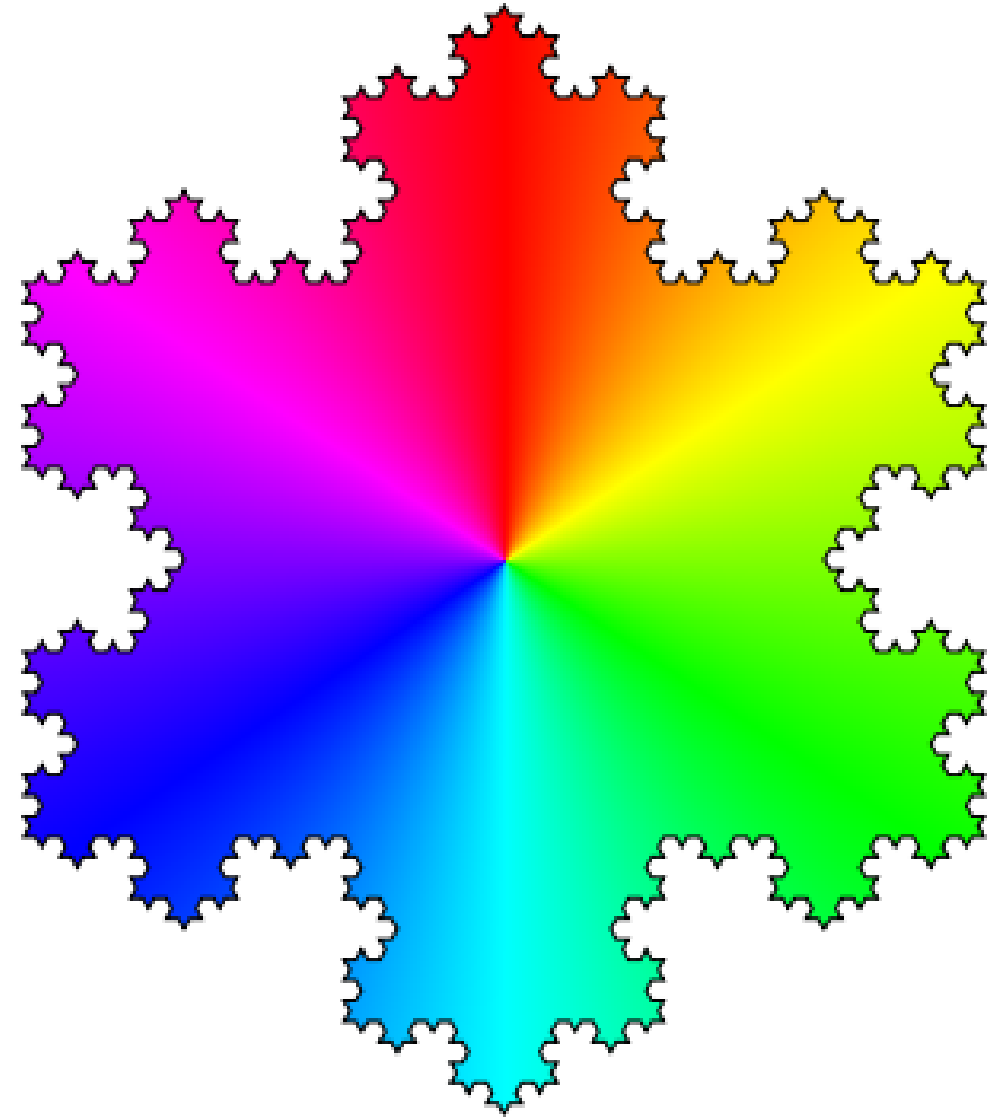
Ben's special snowflake is stolen with three suspects (liars) on the list:

Mike: "I didn't commit the crime."

Leon: "Two people committed the crime."

Ashwin: "Both Leon and Mike participated in the crime."

Who stole Ben's special snowflake?



Ben's special snowflake

JONATHAN'S SOLUTION

Mike says that he didn't commit the crime, which means that he did. Leon's statement is a lie.

Therefore, either one person committed the crime—in this case this would have to be Mike, or all three of them did it.

Ashwin's statement is a lie. Therefore, all three people couldn't have done it together. The crime was committed by Mike.

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VERY BIG THEOREM: On the liars island it is possible to solve any crime.

1. Using our first ideas and puzzles, we deduce that the steps of the BIG THEOREM on the truth-tellers island can be remade into yes/no questions
2. Our steps stay the same





Mainland

Starring: Ashwin and Leon

BEN'S PUZZLE

Holy potato of Joe Biden stolen with four suspects, one of each type, of which two are guilty.

Neil: "I'm a partial truth-teller. Mike is not."

Mike: "I'm a partial truth-teller. Neil is not. He is a scrub though."

Nastia: "I'm a banana. Neil is telling the truth."

Leon: "I like potatoes. Neil is an absolute liar."

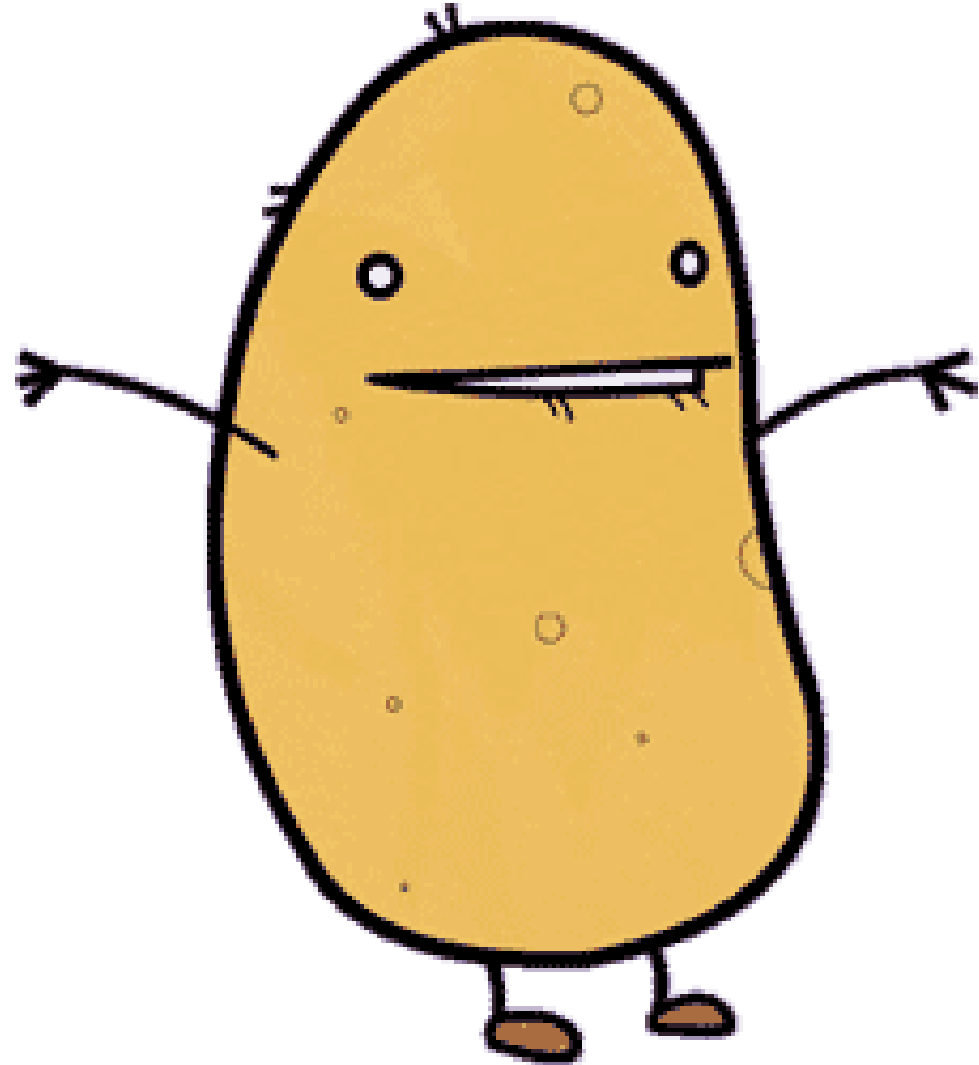
Mike: "I have no idea who did it."

Neil: "I didn't do it. And Leon didn't do it either."

Nastia: "I did it. Leon did not."

Leon: "I did it."

Who stole the holy potato of Joe Biden?



The super holy potato of Joe Biden

BEN'S SOLUTION

Nastia is lying because she cannot be a banana. So Neil is lying as well because Nastia calls Neil a truth-teller. So Nastia and Neil are liars and Mike and Leon are truth-tellers. Leon must have done it since he is a truth-teller and he says "I did it." It also follows that Leon is an absolute truth-teller and Mike is a partial truth-teller. We know that Neil is an absolute liar because Leon, a truth-teller, says so. Neil's "I didn't do it" is a lie, so Neil and Leon did it together. By elimination Nastia is a responsible liar.

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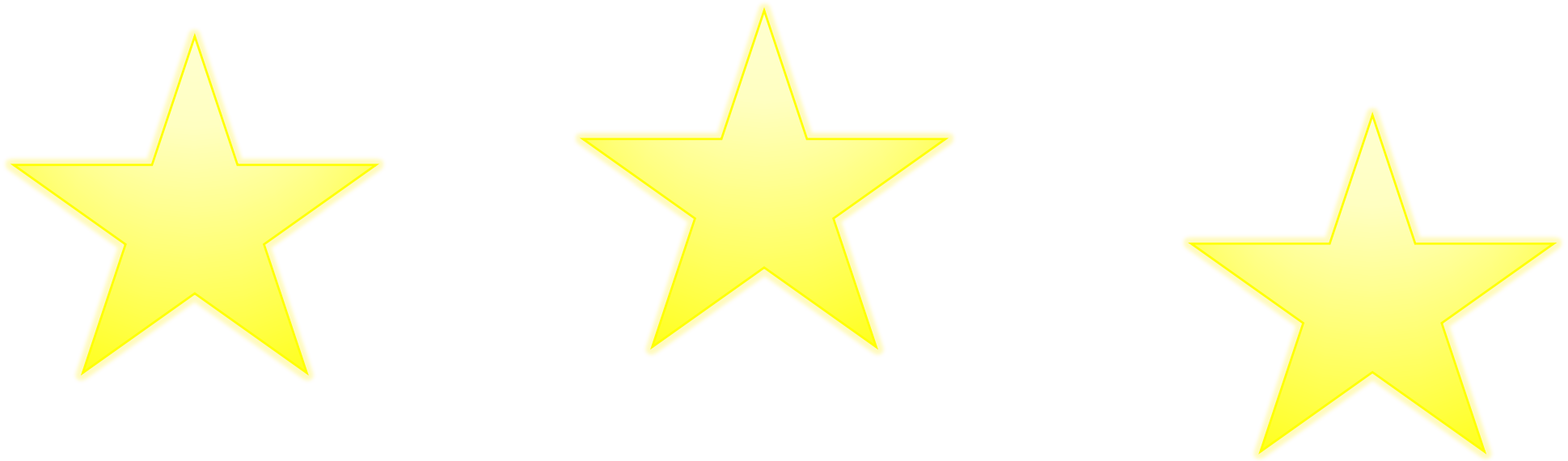
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Nastia: "I did it. Leon did not."

VERY VERY VERY VERY BIG THEOREM: If a crime was committed any number and any type of people it is always possible to solve it.

OR: We can solve any crime in our island nation. Steps:

1. The detective separates all the truth-tellers and the liars by asking an obvious question with the answer known such as "Is my shirt blue?"
2. After the detective deduces who are the truth-tellers and who are the liars, we follow the steps of our two previous big theorems.



THANK **YOU** FOR WATCHING

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