Rotor-Routers

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MIT PRIMES

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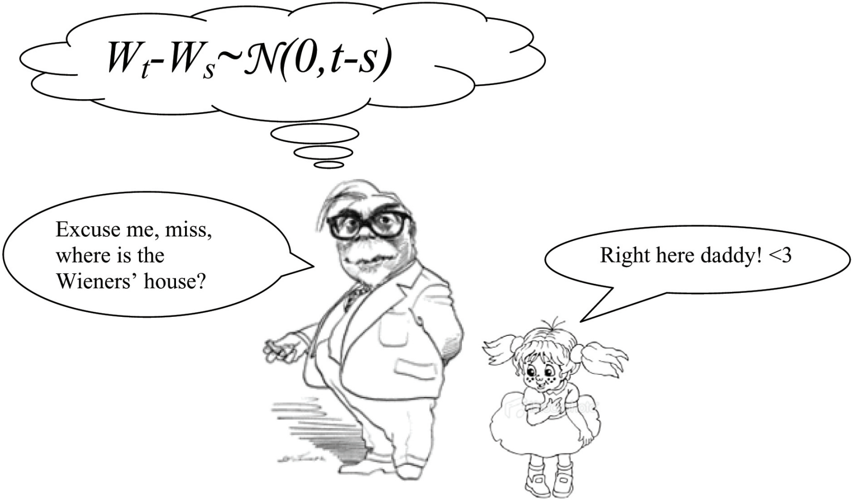
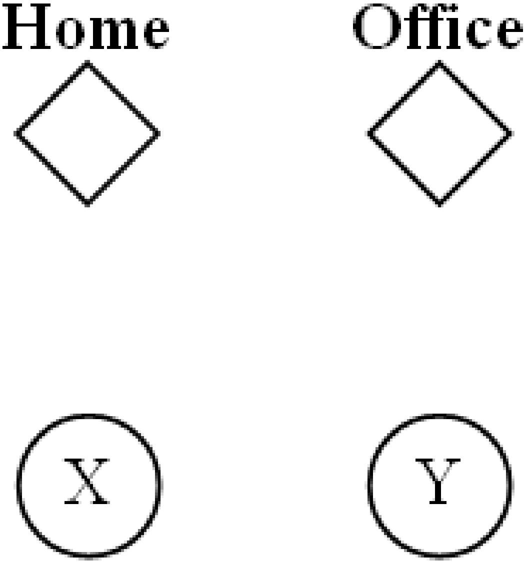
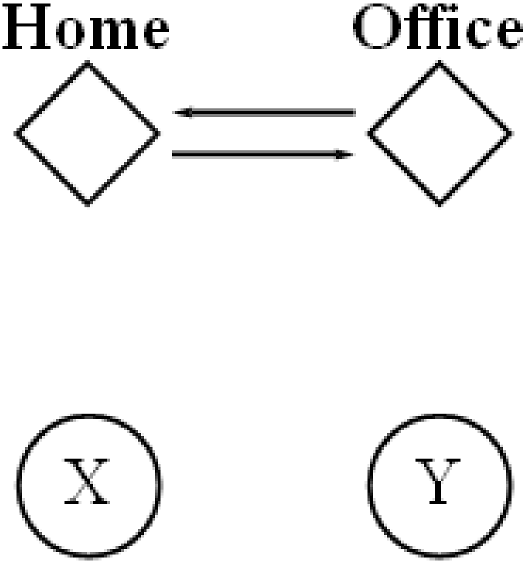
A Problem: The Anterograde Amnesiac



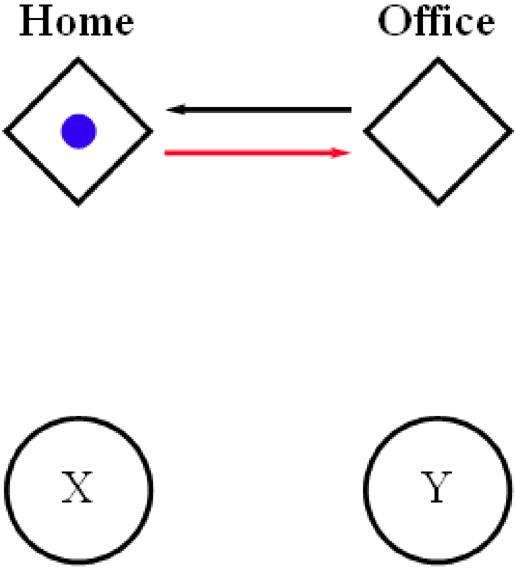
A Problem: The Anterograde Amnesiac



A Solution: Leave Notes



Day 1



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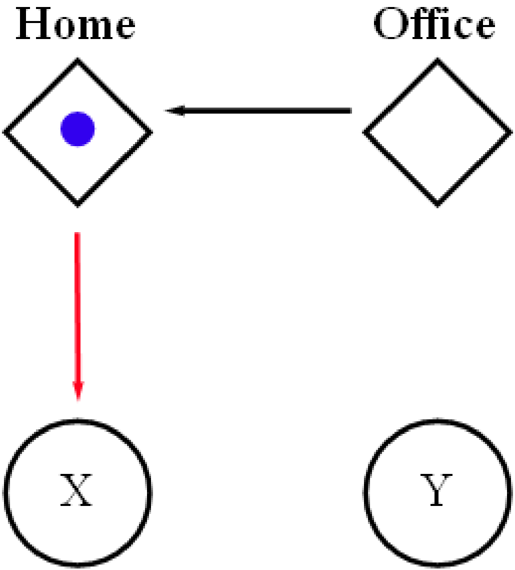

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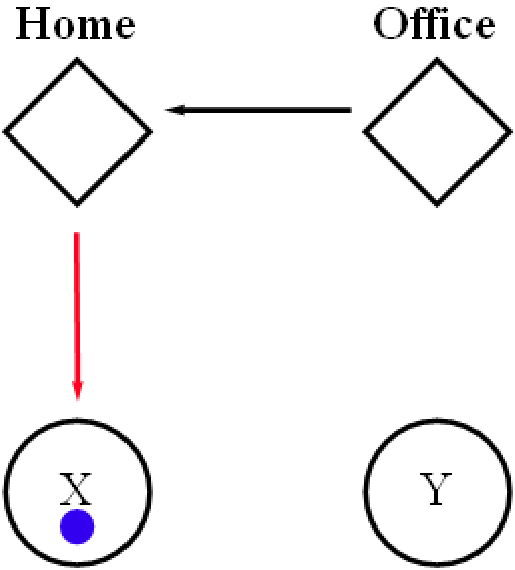
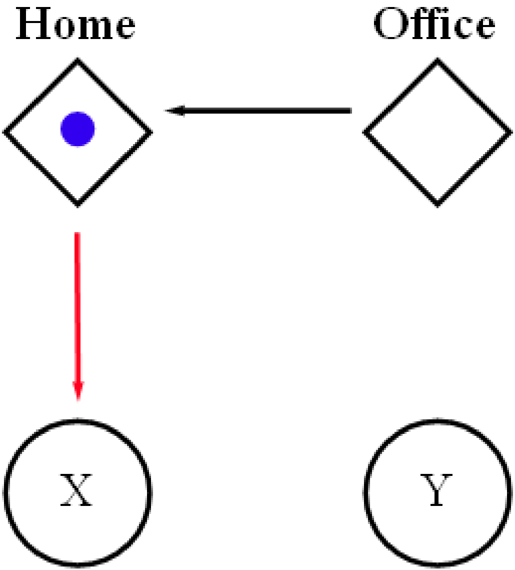


Day 2



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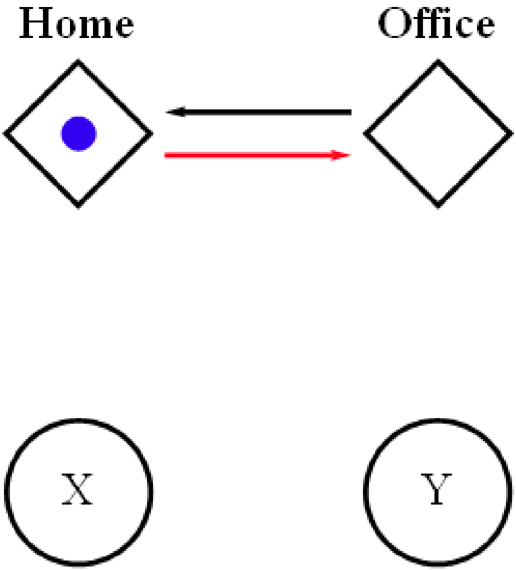

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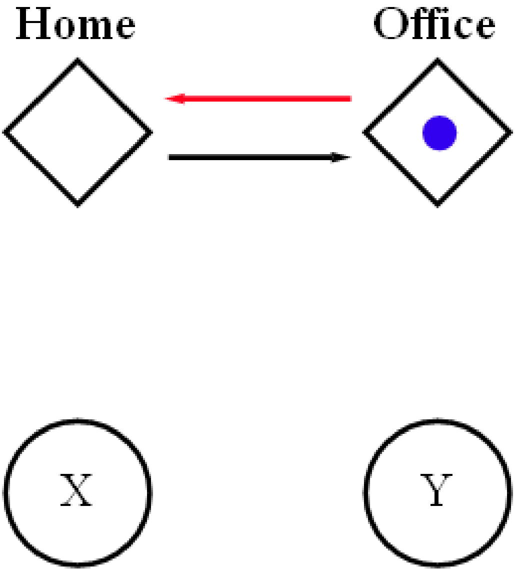

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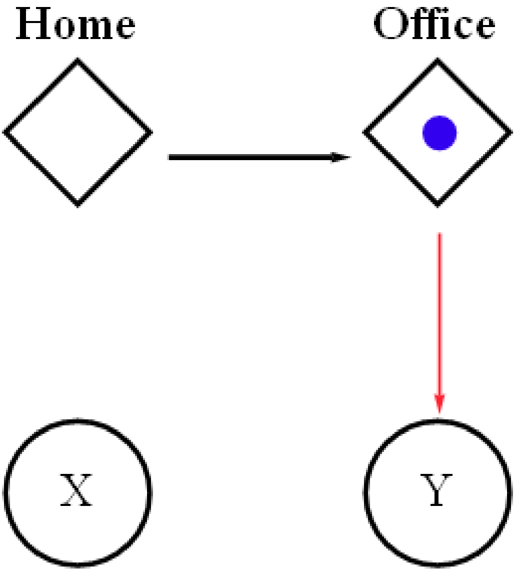

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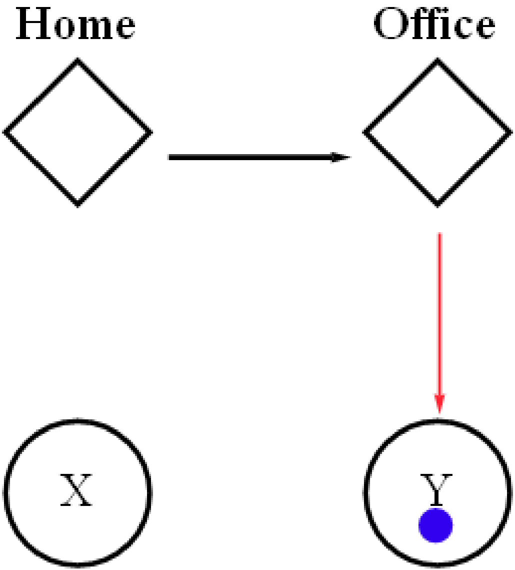
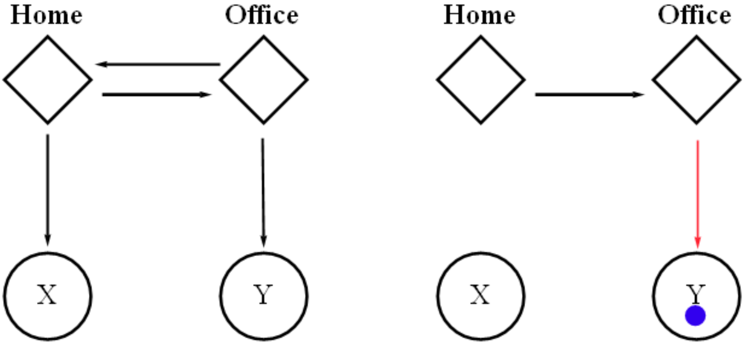


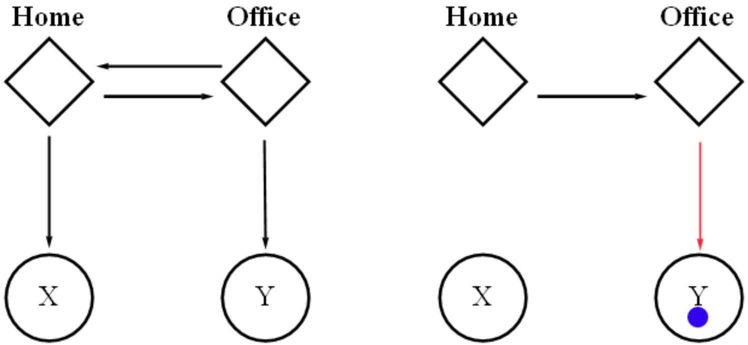
What is a Rotor-Router?

A **rotor-router system** defines a mechanism for travelling around a (directed) graph. At each vertex is a **rotor** that defines the edge the next **particle** entering that vertex will leave by. The rotor consists of a cyclic ordering of the edges emanating from that vertex, some of which may be repeated, and one designated current edge. When a particle enters the node it shifts the current edge and moves out along the new one.



What is a Rotor-Router?

Particles travel through the graph beginning at the designated **source** vertex until they reach one of the **target** vertices, whereupon they leave the system.

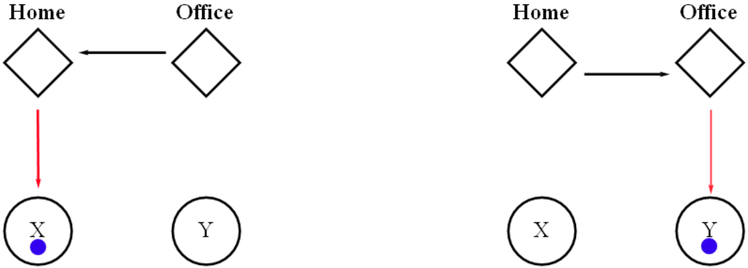


Underlying Graph

Rotor-Router System

What is a Rotor-Router?

In the example, the hitting sequence began XY...



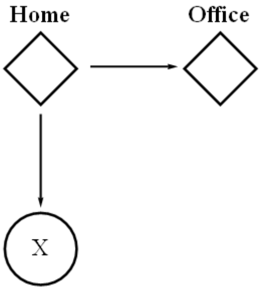
End of Day 1

End of Day 2

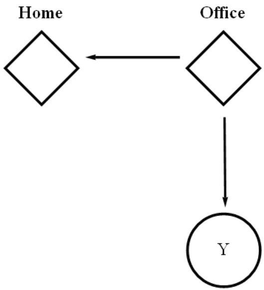
The **hitting sequence** of a rotor system is the sequence of targets hit by particles when they are fed into the system one by one. The hitting sequence is always periodic.

What is a Rotor-Router?

The **rotor type** of a rotor defines the rule by which the rotor cycles through its out-going edges. For instance the rotors in the example are both of type **12**.

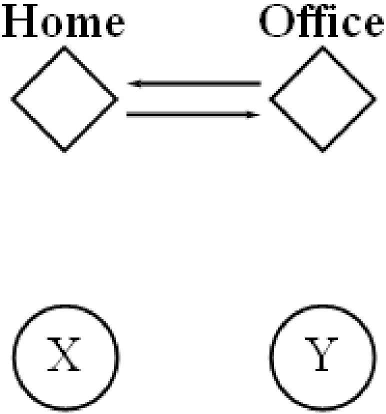


Rotor type of the Home rotor: **12**



Rotor type of the Office rotor: **12**

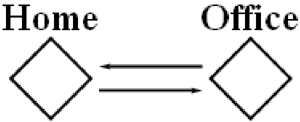
Making a 121 Rotor From 12 Rotors



The hitting sequence of the graph is XYXXYX..., which is of type 121. If a rotor type y can be the hitting sequence of a rotor system in which every rotor is of type x, then we say x can make y.

Which rotors are universal?

Do there exist rotors that can make all other rotors? If so, do there exist rotors that can't?



We can make 121 from 12. Is 12 universal?

Previous Knowledge

12 is Universal

Any other rotor can be made from **12** rotors using a modified binary tree.

Some Rotors are Not Universal

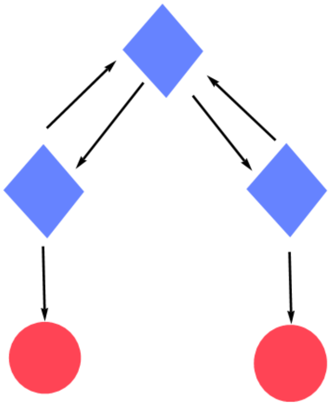
boppy rotor types: all rotors that are either **block-repetitive** or **palindromic**:

1. **block-repetitive** - comes in uniform blocks of the same length. e.g. **1122**
2. **palindromic** - same forwards and backwards. e.g. **121**

Universal	**Not Universal**
12	121
	1122

The Compressor

The **compressor** is a powerful tool working with two-state rotor types. To apply it, replace each vertex with the same rotor type, and find the hitting sequence. After one application of the compressor, a rotor type becomes balanced. Further applications do not increase its period.



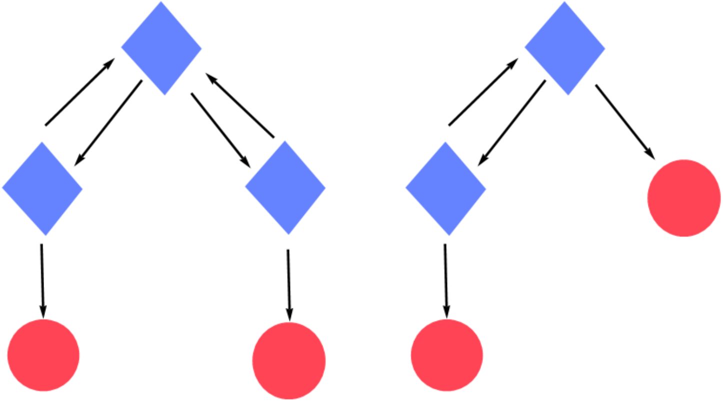
The Compressor

Using a computer search, we found that **all but 274 of the unboppy rotor types of length up to 17 are universal** by directly applying the compressor. This is out of a total of more than 100,000 possible rotor types. In particular, this settled the original question of whether **11222** is universal, proposed by Jim Propp.

Len	# Undecided
2	0
3	0
4	0
5	0
6	2
7	0
8	5
9	9
10	22
11	0
12	36
13	0
14	12
15	52
16	136
17	0

The Compressor

For balanced rotors, we proved by supplementing the compressor with another simple configuration that **all balanced, unboppy rotors of length up to 23 are universal.** There were 10 problematic cases for length 24.



Reductions

The Reduction Theorem

If every unboppy two-state rotor is universal, then all unboppy rotors are universal.

Rotor Type:
111333332222 $\rightarrow$ unboppy
Reductions:
111222222222

Reductions

The Reduction Theorem

If every unboppy two-state rotor is universal, then all unboppy rotors are universal.

Rotor Type:
111333332222 $\rightarrow$ unboppy
Reductions:
111222222222
111111112222

Reductions

The Reduction Theorem

If every unboppy two-state rotor is universal, then all unboppy rotors are universal.

Rotor Type:
111333332222 $\rightarrow$ unboppy
Reductions:
111222222222
111111112222
111333331111 $\rightarrow$ unboppy

Paths for Future Research

1. The **compressor still meets roadblocks** at some fixed points and cycles. Can we find a different configuration that can fix this problem?
2. We conjecture from empirical data that the compressor alone is sufficient to prove the universality of all unboppy rotors of **prime length.**
3. Almost all rotor systems will exponentially increase a rotor's length, unlike the compressor. Is there a measure for this ability to **keep rotor lengths small**? Can we find other such systems with **even more power**?

Acknowledgements

Thanks to:

1. **Tanya Khovanova**, for her big-picture insight, awesome mathematical experience, humor, and guidance.
2. **James Propp**, for inventing rotor-routers, proposing the project, and providing fun and descriptive nomenclature.
3. **PRIMES staff**, for their impressive organizational abilities and for making this experience possible.
4. **My Parents**, for providing transportation and other forms of support.