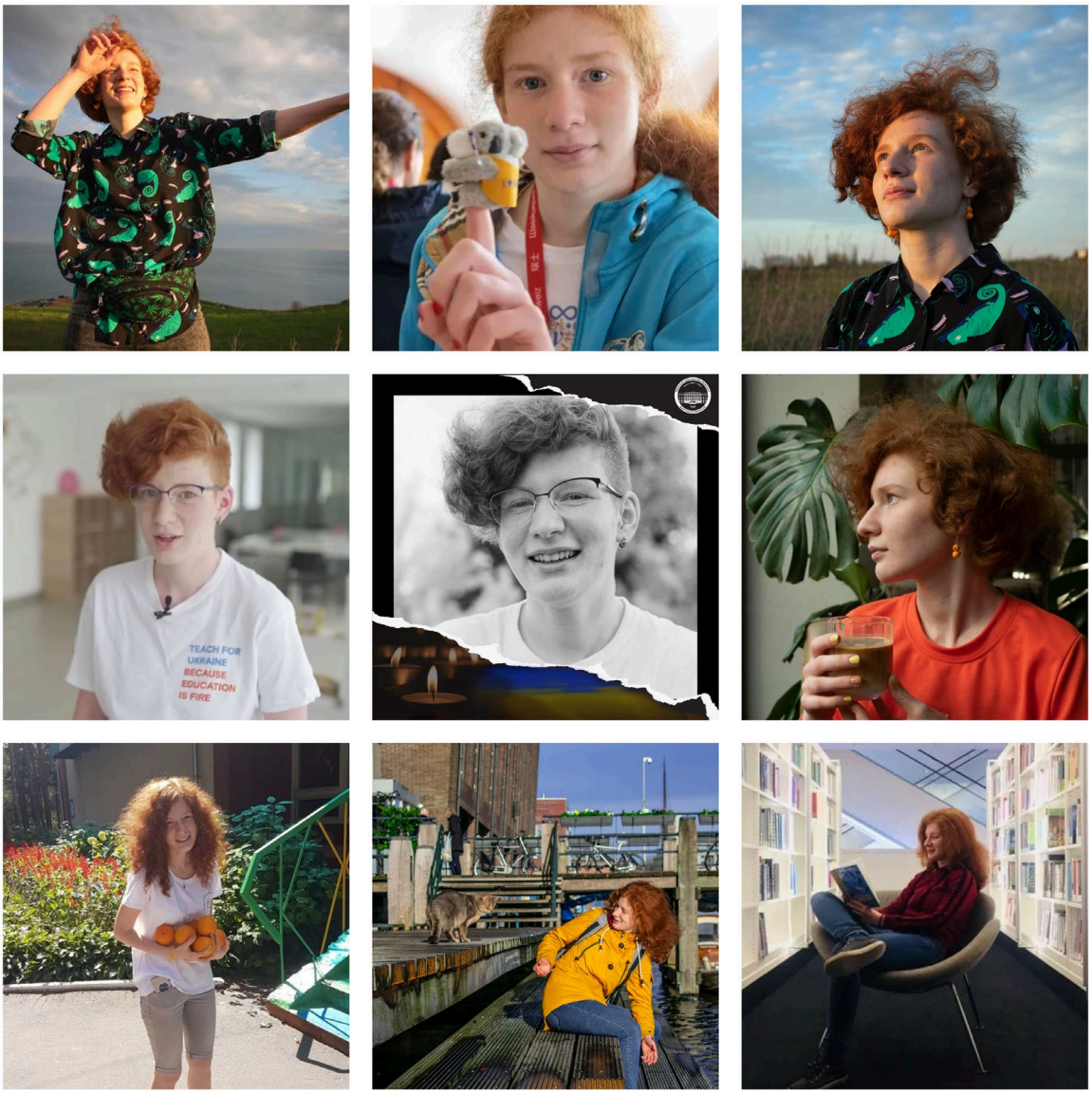
Program for Research in Mathematics, Engineering and Science



First Annual
Yulia's Dream
Virtual Conference
May 14-15, 2023

First Annual Yulia's Dream Conference

Sunday, May 14

9:00 am (Boston) [16:00 (Kyiv)]: Welcoming Remarks

- Prof. Pavel Etingof, Yulia's Dream Chief Research Advisor
- Dr. Slava Gerovitch, Yulia's Dream Program Director

9:15-11:10 am (Boston) [16:15-18:10 (Kyiv)]: Session 1

- Ivan Bortnovskyi, Vadym Pashkovskyi, and Ihor Pylaiev, “Exploration of Grothendieck-Teichmueller (GT) shadows for the dihedral poset” (mentor Prof. Vasily Dolgushev, Temple University)
- Yehor Avdieiev, “Affine standard Lyndon words” (mentor Prof. Alexander Tsymbaliuk, Purdue University)
- Dmytro Antonovych, Viktor Makozyuk, and Vladyslav Tysiachnyi, “Matching of frames in open Jacobi diagrams and chord diagrams spaces” (mentor Dr. Mykola Semenyakin, Perimeter Institute for Theoretical Physics)
- Alexander Borodin, Martin Leshko, and Marina Spektrova, “Combinatorial Hikita Conjecture” (mentor Dr. Andrei Ionov, Boston College)
- Glib Dmytriiev, Masha Matsiiako, and Pavlo Shekhet, “Lagrange’s Theorem” (mentor Dr. Vanya Yakovlev)

Monday, May 15

8:30-11:00 am (Boston) [15:30-18:00 (Kyiv)]: Session 2

- Artem Chernii, Davyd Poplyonkin, and Andrii Smutchak, “Weighing the Odds: A Probabilistic Approach to Coin Weighing” (mentor Anton Trygub, MIT)
- Constantine Bulavenko and Yaroslav Molybog, “Distinguishing Knots: The Jones Polynomial and Integer Invariants” (mentor Julius Baldauf, MIT)
- Ivan Balashov and Maksym Torianyk, “Knots and Reidemeister Theorem” (mentor Roman Krutowski, University of California, Los Angeles)
- Yaroslav Ibragimov, Dmytro Kulish, and Dang Minh Kong, “The Generalized Stokes’ Theorem and its Applications” (mentor Kyrylo Muliarchyk, University of Texas at Austin)

- Volodymyr Chub, Maksym Petrenko, and Stanislav Surmylo, “Riemann integral in higher dimensions” (mentor Maksym Chaudkhari, University of Texas at Austin)
- Sophia Breslavets, Igor Goldanskii, and Denys Honcharov, “Proof of Abel’s theorem” (mentor Elizaveta Nesterova, Technion – Israel Institute of Technology)
- Semen Andriets and Borys Holikov, “Partial orderings of minors in the positive Grassmannian” (mentor Yelena Mandelshtam, University of California, Berkeley)
- Severyn Khomych, Nazar Korniichuk, and Kostiantyn Molokanov, “The Pak-Postnikov and Naruse skew hook length formulas: A new proof” (mentor Prof. Darij Grinberg, Drexel University)

11:00 am (Boston) [18:00 (Kyiv)]: Session 3

- Prof. Andrey Gogolev (Ohio State University), Welcome from the International Centre for Mathematics in Ukraine (ICMU)
- Prof. Maryna Viazovska (École Polytechnique Fédérale de Lausanne; winner of the Fields Medal 2022), “The story of almost impossible Leech lattice”

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Sunday, May 14

Session 1

Ivan Bortnovskyi and Vadym Pashkovskyi
Exploration of Grothendieck-Teichmueller (GT) shadows for the dihedral poset
Mentor: Prof. Vasily Dolgushev, Temple University

Grothendieck-Teichmueller group was introduced by Vladimir Drinfeld in Section 4 of *"On quasitriangular quasi-Hopf algebras and on a group that is closely connected with* $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$*"*. It is closely related and possibly isomorphic to the absolute Galois group of the rational numbers $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$. In our presentation we describe the groupoid GTSh whose objects are certain finite index normal subgroups of the Artin braid group B_3 on 3 strands. The morphisms of this groupoid can be thought of as of approximations of the Grothendieck-Teichmueller group. We also consider the dihedral poset Dih of kernels of certain homomorphisms from the pure braid group PB_3 to the product of dihedral groups D_n^3 (for all $n \geq 3$) and explore the groupoid GTSh in relation to the elements of the poset Dih.

Yehor Avdieiev
Affine standard Lyndon words
Mentor: Prof. Alexander Tsymbaliuk, Purdue University

We generalize an algorithm of Leclerc describing explicitly the bijection of Lalonde-Ram from finite to affine Lie algebras. In type $A_n^{(1)}$, we compute all affine standard Lyndon words for the "standard order" of the simple roots. We also write a Python code that counts affine standard Lyndon words for all types and any order.

Dmytro Antonovych, Viktor Makozyuk, and Vladyslav Tysiachnyi
Matching of frames in open Jacobi diagrams and chord diagrams spaces
Mentor: Dr. Mykola Semenyakin, Perimeter Institute for Theoretical Physics

The talk will be devoted to one conjecture related to the "frames" of open Jacobi diagrams and chord diagrams, which are uni-trivalent graphs with "orientation" structure and trivalent graphs with the structure of the chosen Hamilton cycle respectively. The "frame" operation maps both classes to just trivalent graphs. The conjecture relates frames of the basis sets for the factor-spaces of vector spaces of both types of diagrams, modulus special relations. We will present some examples of analytic checks of the conjecture and also show some results obtained using the self-written code.

Alexander Borodin, Martin Leshko, and Marina Spektrova
Combinatorial Hikita Conjecture
Mentor: Dr. Andrei Ionov, Boston College

The cells in the Hecke algebra of symmetric group can be described with the Robinson-Schensted algorithm. The problems of describing these cells in other cases as well as investigating the longest and shortest elements in them are motivated by geometry and algebra. In our talk, we will briefly review the preliminaries and touch on the main questions we have worked on.

Glib Dmytriiev, Masha Matsiiako, and Pavlo Shekhet
Lagrange's Theorem
Mentor: Dr. Vanya Yakovlev

We are pleased to present our project about Lagrange's Theorem, which focuses on various methods to prove this important theorem. Also, we solved a few related problems. Our investigation includes ways to use Lagrange's Theorem to prove several classical theorems from number theory. So our project shows the versatility, significance, and practical relevance of Lagrange's Theorem across different mathematical topics as a fundamental tool for problem-solving.

Monday, May 15

Session 2

Artem Chernii, Davyd Poplyonkin, and Andrii Smutchak
Weighing the Odds: A Probabilistic Approach to Coin Weighing
Mentor: Anton Trygub, MIT

In this presentation, we explore the problem of identifying the weight of every coin among a set of n coins, each with one of two known weights. Using probabilistic methods, we establish both upper and lower bounds on the minimum number of weighings required to solve this problem, and discuss the broader applications of probabilistic methods in combinatorics.

Constantine Bulavenko and Yaroslav Molybog
Distinguishing Knots: The Jones Polynomial and Integer Invariants
Mentor: Julius Baldauf, MIT

In this presentation, we discuss the use of knot invariants to distinguish knots. We begin by introducing a simple invariant, tricolorability, and then the Jones polynomial, a very strong knot invariant. The Jones polynomial is difficult to compute, growing exponentially with the number of crossings in a knot projection. To overcome this problem, we also introduce integer invariants such as the linking number and enhanced linking number. These invariants are weaker than the Jones polynomial, but they are easier to compute, making them more efficient for knot and link projections with a large number of crossings.

Ivan Balashov and Maksym Torianyk
Knots and Reidemeister Theorem
Mentor: Roman Krutowski, University of California, Los Angeles

The talk consists of an introduction to the knot theory review and the Reidemeister theorem with proof. Initially discovered in 1926–1927, the theorem states a connection between diagrams of equivalent knots. Its simplicity makes it one of the most useful tools for analyzing those. Consequently, most knot invariants based on their diagrams were created using it. Some of them are included in the talk.

Yaroslav Ibragimov, Dmytro Kulish, and Dang Minh Kong
The Generalized Stokes' Theorem and its Applications
Mentor: Kyrylo Muliarchyk, University of Texas at Austin

The generalized Stokes theorem is a powerful tool in multivariable calculus that relates integrals over a boundary to integrals over the interior of a region. It generalizes the fundamental theorem of calculus for higher dimensions. In this talk, we will present the generalized Stokes theorem and its applications in various areas of mathematics, physics, and engineering. We will discuss the key concepts needed to understand the theorem, such as differential forms, manifolds, and their boundaries. Finally, we will deduce classical Green's theorem and divergence theorem from it.

Volodymyr Chub, Maksym Petrenko, and Stanislav Surmylo
Riemann integral in higher dimensions
Mentor: Maksym Chaudkhari, University of Texas at Austin

We will give a brief overview of the Riemann integral in higher dimensions. In particular, we will discuss the integrability criterion, Fubini's theorem, and change of variables theorem. We will also explain some of the limitations of Riemann's definition of the integral.

Sophia Breslavets, Igor Goldanskii, and Denys Honcharov
Proof of Abel's theorem
Mentor: Elizaveta Nesterova, Technion – Israel Institute of Technology

In our talk we will explore Abel's theorem in a way everyone can understand using Kirillov's book as a guide. Denys will discuss group properties and show that the group of permutations of more that 4 elements is not soluble. Sophia will explain concepts related to Riemann surfaces and monodromy groups in an easy-to-understand way. Then, Igor will demonstrate that the monodromy group of a function expressible by radicals is soluble. Together, we will prove that the monodromy group of a polynomial is not soluble, tying all these ideas together.

Semen Andriets and Borys Holikov
Partial orderings of minors in the positive Grassmannian
Mentor: Yelena Mandelshtam, University of California, Berkeley

The totally positive Grassmannian is in bijection with $k \times n$ matrices (where $k \leq n$) such that all of the maximal minors, or determinants of the $k \times k$ sub-matrices, are positive. In this talk, we will discuss the structure of minors in the positive Grassmannian. It has been shown that sets of largest (by value) minors correspond to maximal sorted sets, and that a choice of largest minors induces a poset structure on all of the minors. However, beyond the second level of the poset, in most cases, this structure is still only conjectured. We investigate this structure and will talk about some of the ways that we are trying to prove the conjecture.

Severyn Khomych, Nazar Korniichuk, and Kostiantyn Molokanov

The Pak-Postnikov and Naruse skew hook length formulas: A new proof

Mentor: Prof. Darij Grinberg, Drexel University

The hook length formula is a classical result in enumerative combinatorics, expressing the number of standard Young tableaux of a given partition shape as a single fraction. In recent years, two generalizations of this formula have emerged: one by Pak and Postnikov, replacing the number by a (rational) generating function, and one by Naruse, which generalizes the setting from a partition to a skew partition. Both of these generalizations appear to lie significantly deeper, with no simple proofs known. We combine them into a generating-function identity for skew partitions, and prove it in a fairly elementary way using recursion, determinants and the Lindström–Gessel–Viennot lemma.