

Topologie de l'ADN

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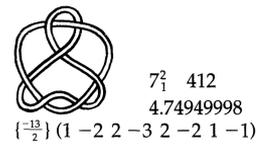
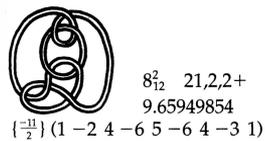
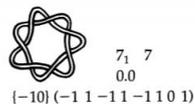
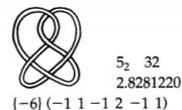
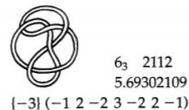
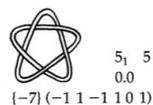
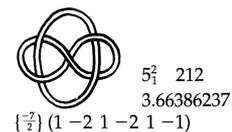
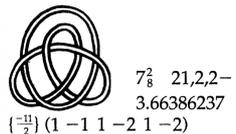
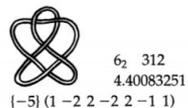
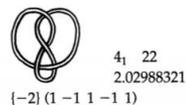
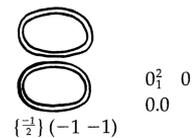
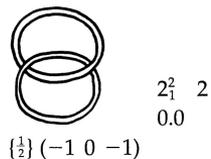
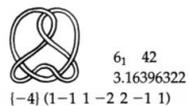
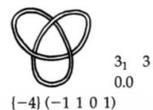
Définitions

Nœud : un plongement d'un cercle dans \mathbf{R}^3 considéré à des déformations continues près.

Entrelacs: plongement de plusieurs cercles disjoints dans l'espace, à déformation près.

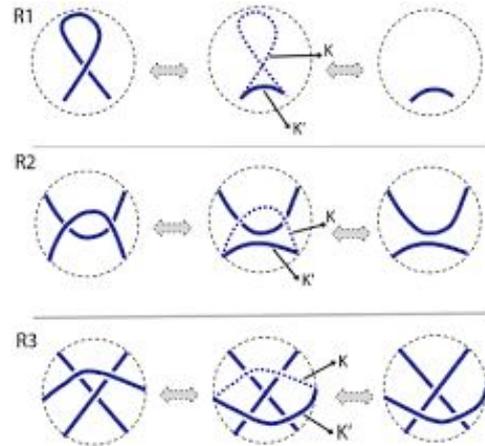
Invariant : Une quantité déterminée pour un noeud qui ne change pas quand on le déforme.

Exemples de noeuds et entrelacs

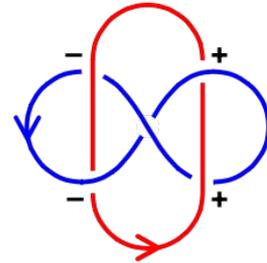


Théorème de Reidemeister

Toute déformation peut être représentée comme une composition de ces trois mouvements.



Linking number



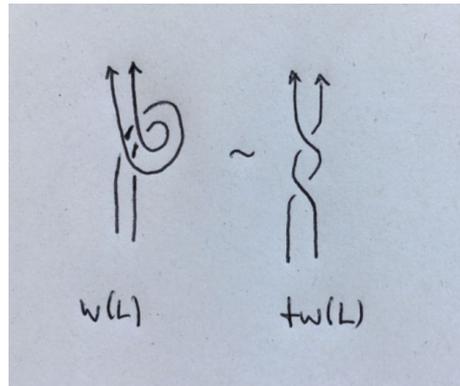
Motivation biologique

L'ADN est un entrelacs à deux composantes /noeud ruban.

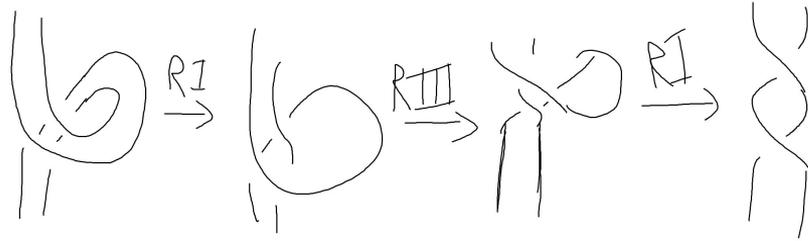
Certains phénomènes biologiques (réplication, transcription, combinaison) agissent sur la topologie de l'ADN par des enzymes, les topoisomérases.



Writhe and twist



Twist = writhe

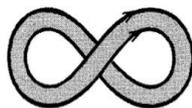


Linking number = twist + writhe



$$\begin{aligned} \text{Lk}(R) &= +1 \\ \text{Tw}(R) &= +1 \\ \text{Wr}(R) &= 0 \end{aligned}$$

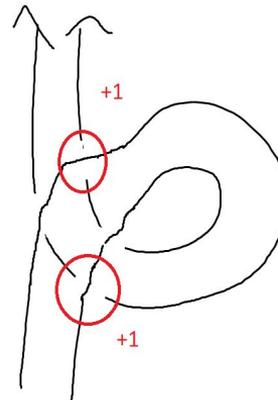
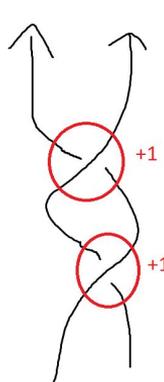
a



$$\begin{aligned} \text{Lk}(R) &= +1 \\ \text{Tw}(R) &= 0 \\ \text{Wr}(R) &= +1 \end{aligned}$$

b

7.9 Twist, writhe, and linking number.



Supercoiling

$$\begin{aligned}lk'(L) &= lk(L) \\ \Rightarrow tw'(L) &= tw(L) + 1 \\ w'(L) &= w(L) - 1\end{aligned}$$

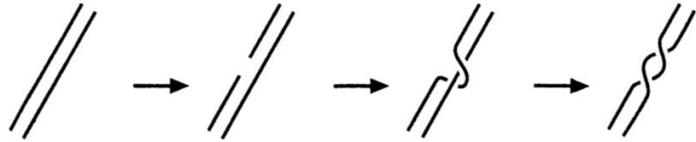


Relaxed phone cord

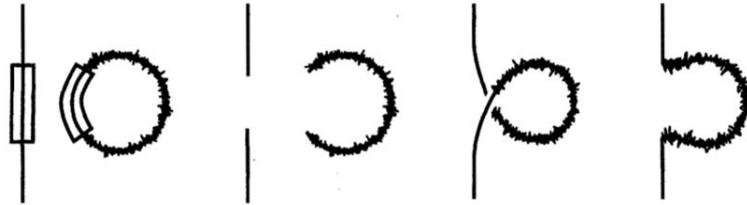


Supercoiled phone cord
after adding twists

Actions des enzymes



7.13 An enzyme adds a full twist to a DNA molecule.



7.21 Int enzyme inserts bacteriophage λ viral DNA.

Des caractéristiques physiques différentes

- Topologiquement identiques mais caractéristiques énergétiques différentes
- Peut être mis en valeur par une expérience
- Interactions différentes pour les molécules supercoilées/non supercoilées



Figure 7.14 Supercoiled molecules move faster through the gel.

Bibliographie

- Slides 3, 10, 11, 12 et 13: Référence images: Colin C. Adams, *The Knot Book*, American Mathematical Society, Providence, Rhode Island, “Mathematical Subject Classification”, 2000, pp 182 ,187 ,189 , 190, 194, 280-290
- Slide 4: Référence image: <https://ieeexplore.ieee.org> ; IEEE Transactions on Visualization and Computer Graphics (Volume: 18, Issue: 12, Dec. 2012); Hui Zhang, Jianguang Weng, Lin Jing, Yiwen Zhong
- Slide 5: Référence images, de gauche à droite: ResearchGate, Marcos Marino, https://www.researchgate.net/figure/When-computing-the-linking-number-of-two-knots-the-crossings-are-assigned-a-sign-1-as_fig13_2060101; Référence image: Wikipédia, Linking number, https://en.wikipedia.org/wiki/Linking_number
- Slide 6: Référence image: France Culture, <https://www.franceculture.fr/emissions/la-methode-scientifique/la-methode-scientifique-mercredi-10-janvier-2018>

Remerciements

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