Cayley’s Formula

Primes-Switzerland
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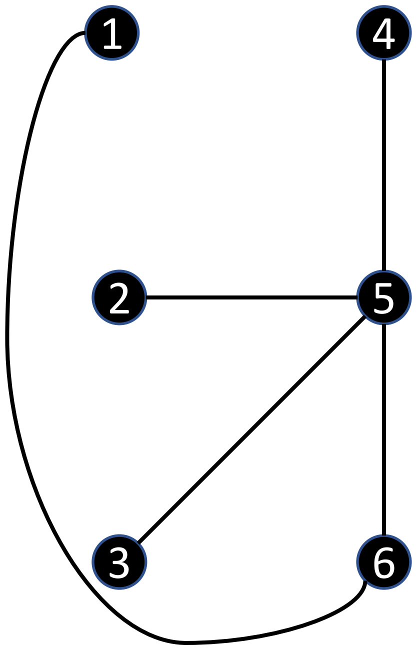
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4
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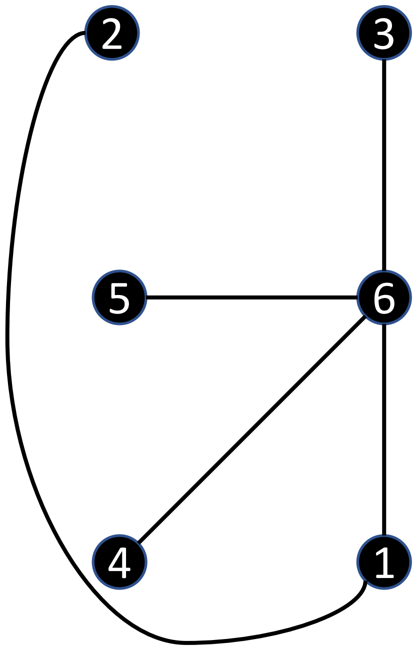
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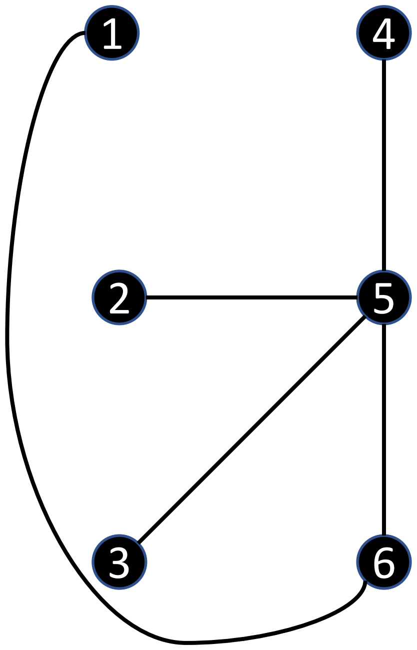
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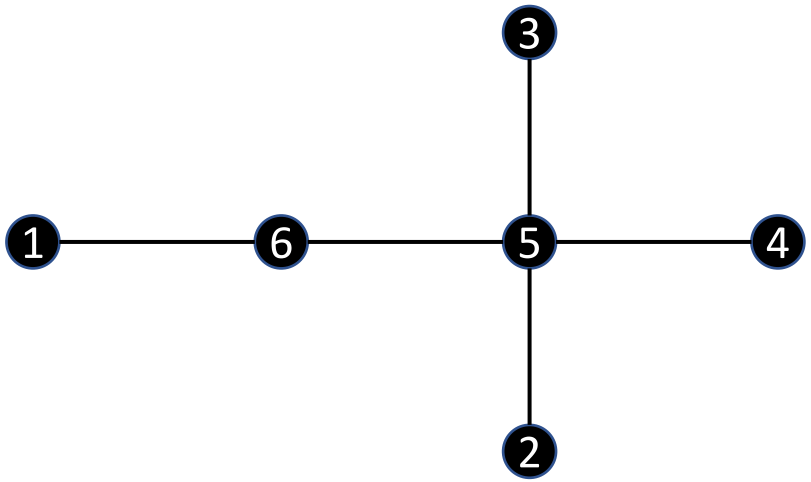
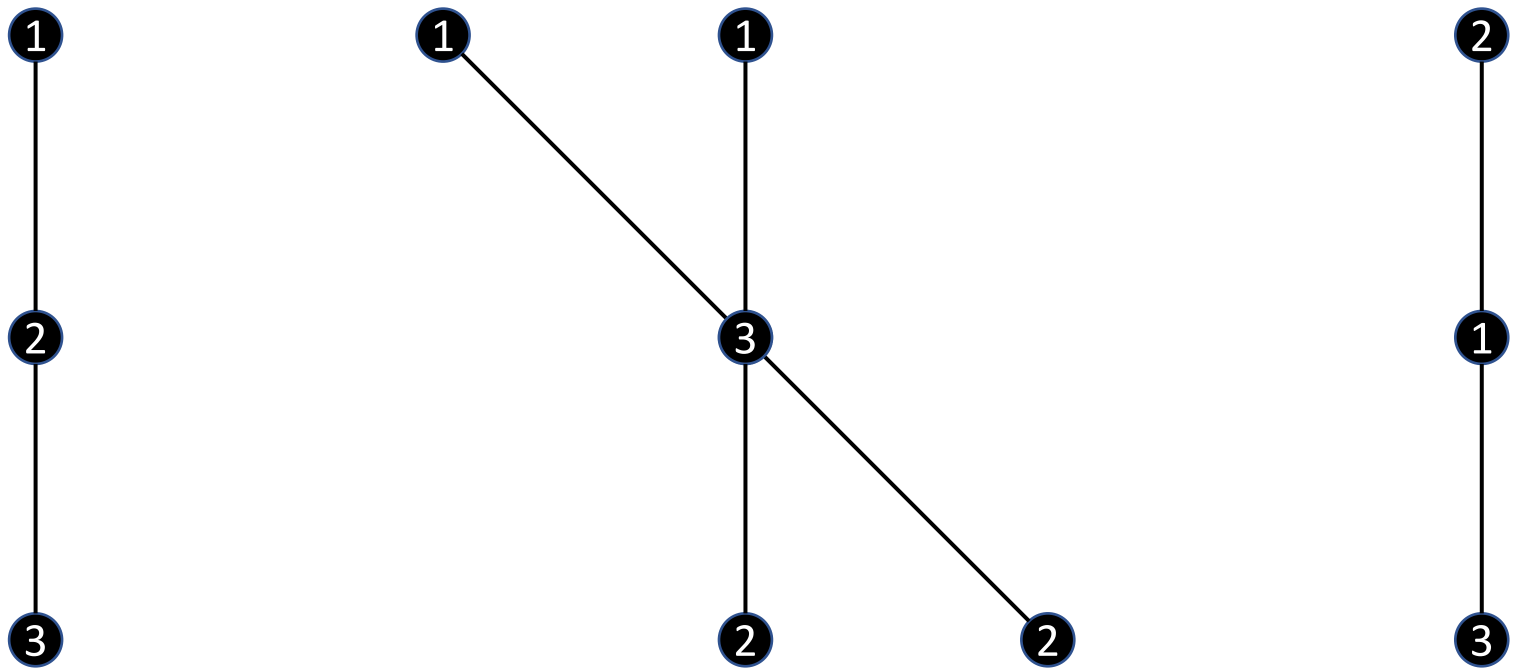

3
1
6
5
4
2


1
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1
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2
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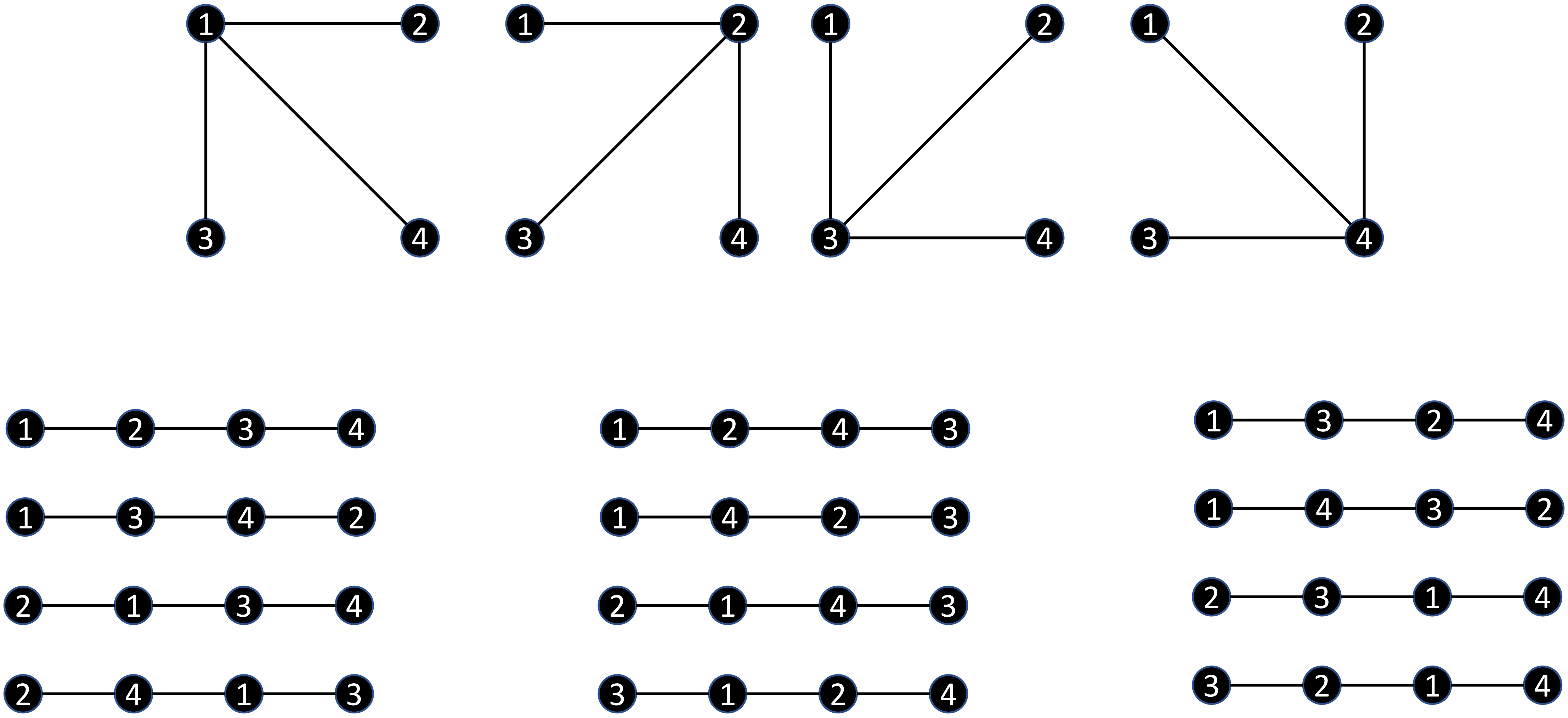
2
1
3



1 2 3 4
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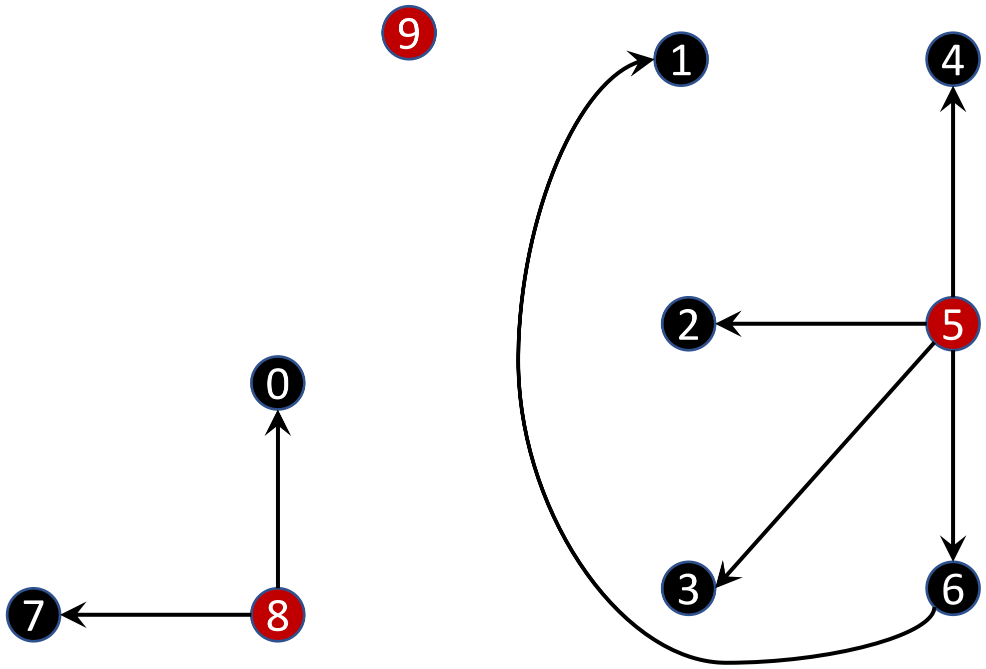
1 3 2 4
1 3 4 2
1 4 2 3
1 4 3 2
2 1 3 4
2 1 4 3
2 3 1 4
2 4 1 3
3 1 2 4
3 2 1 4

n	A_n
1	1
2	1
3	3
4	16

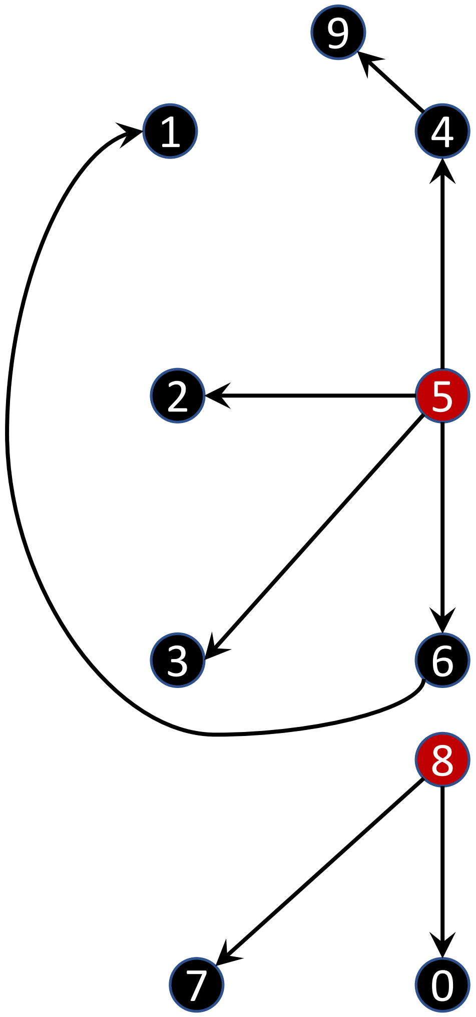
Cayley's Formula:

$$A_n = n^{n-2}$$

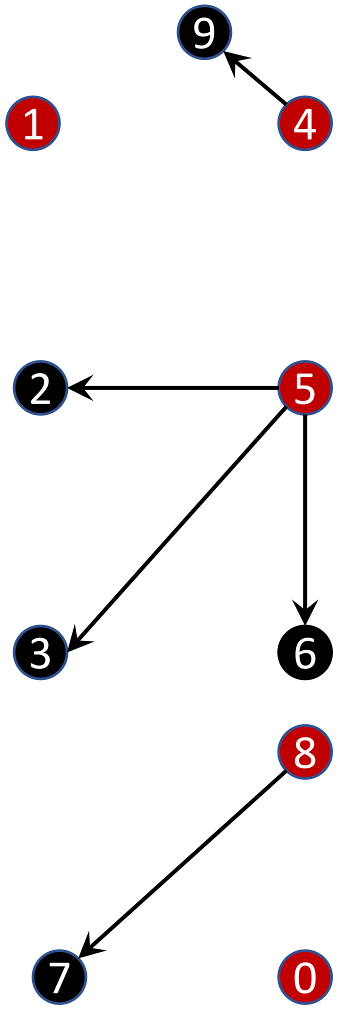


A rooted forest, viewed as a directed graph

- For each component, one vertex is called a root.
- Every edge is directed away from the root.

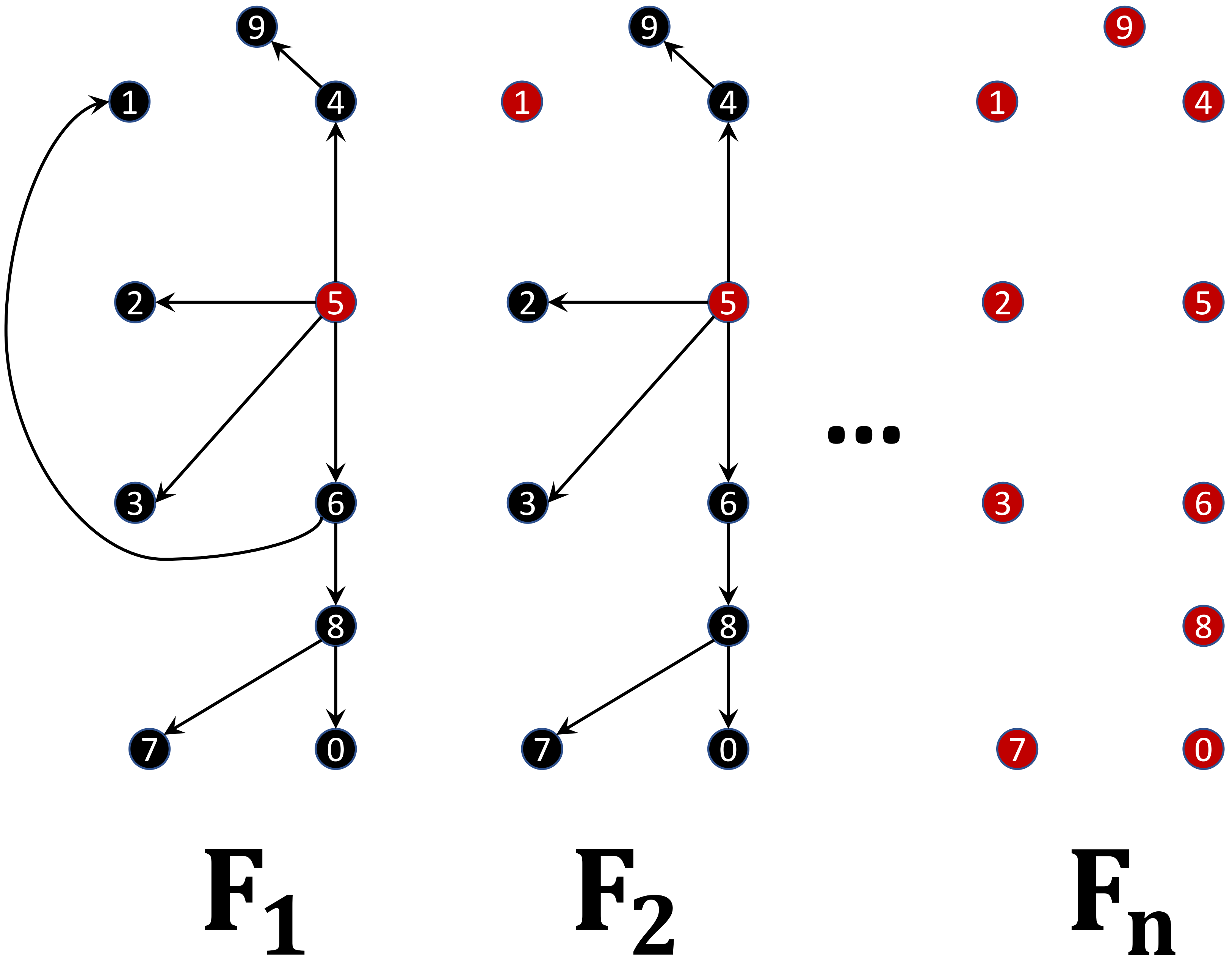


F



F’

F contains F’
If, in F’, an edge starts at vertex x and ends at vertex y, there also is an edge from x to y in F.

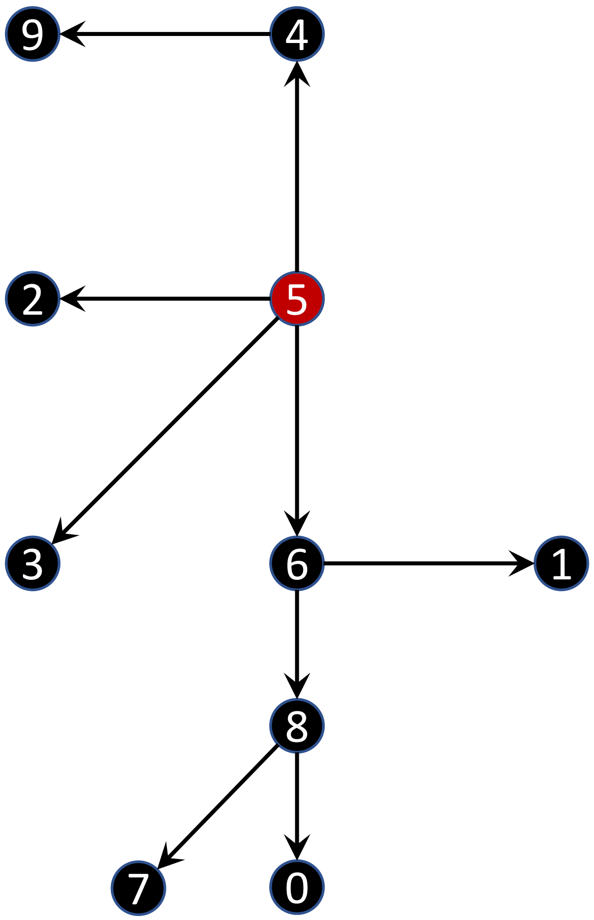


A refining sequence $(F_1,\ldots,F_n)$

- Each forest F_i contains F_{i+1}.
- Each forest F_i has exactly i components.

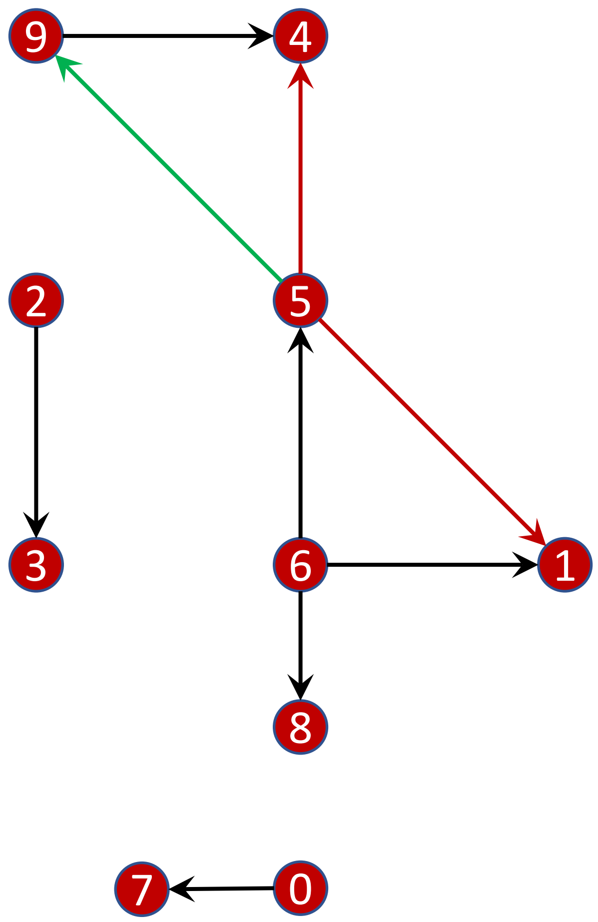
N: #rooted trees on n vertices

N*: #refinig sequences $(F_1,\ldots,F_n)$



$$N^* = N(n-1)!$$

$F_{[illegible]-1}$



F_{in-11}

N: #rooted trees on n vertices
N*: #refining sequences $(F_1, \ldots, F_n)$

$$N^* = n(n-1) * n(n-2) \ldots n * 1$$

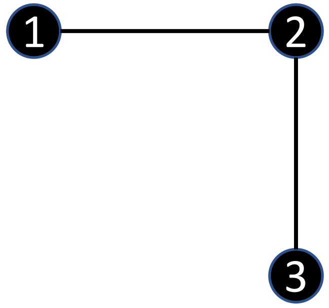
$$N^* = n^{n-1}(n-1)!$$

$$N^* = N(n-1)!$$

$$N = n^{n-1}$$

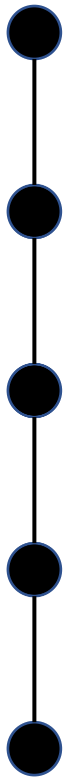
$$N = A_n * n$$

$$A_n = n^{n-2}$$



A_5

$A_5 = 5^3 = 125$



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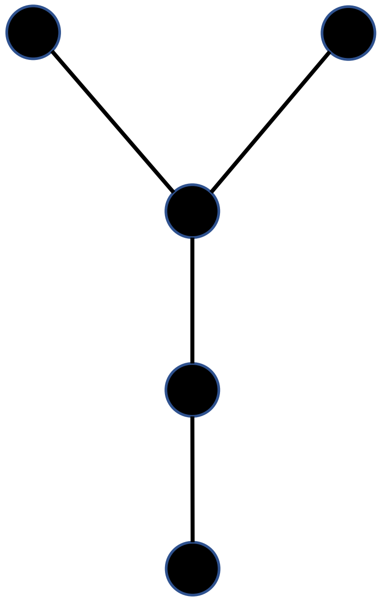

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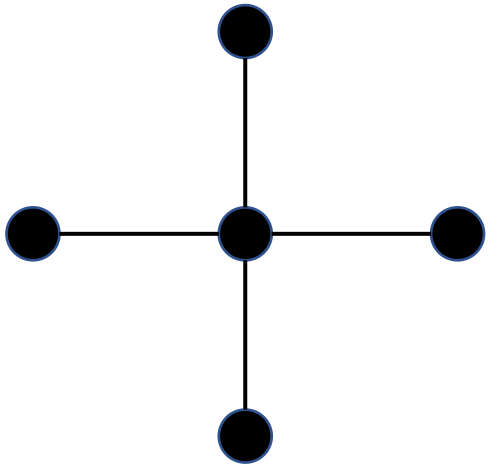

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