Abstracts

PuMa GraSS

Fall 2018

On Hochster's Stone Duality

Peter Haine

One of the first things that we learn about the absolute Galois group $Gal(K^{sep}/K)$ of a field K is that $Gal(K^{sep}/K)$ is more than just a group — it has a natural topology that makes it into a profinite group. This topology is extremely useful, and comes from the classical Stone duality theorem which identifies pro-objects in the category of finite sets with profinite or Stone spaces, i.e., compact Hausdorff totally disconnected topological spaces. Lesser-known is Hochster's generalization of the Stone duality theorem which identifies pro-objects in the category of finite posets with spectral topological spaces, a certain class of quasi-compact and quasi-separated topological spaces. The name 'spectral' comes from a very surprising result of Hochster: a topological space X is spectral if and only if X is homeomorphic to the underlying Zariski space of the spectrum of some commutative ring. This condition turns out to also be equivalent to saying that X is the underlying Zariski space of some quasi-compact quasi-separated scheme.

In this talk we'll explain what all of the above terms mean and discuss Hochster's generalization of Stone duality. We'll also discuss a nontrivial duality on the category of spectral topological spaces that Hochster's theorem provides. If time permits, we'll mention some recent work (which uses Hochster's theorem as an input) on how to construct a 'purely algebraic' invariant of schemes that generalizes the absolute Galois group of a field and is a complete invariant of certain 'nice' schemes in characteristic 0.

Scalar curvature and Positive Mass Theorem

Ao Sun

Positive energy theorem, which is more commonly known as the positive mass theorem, is one of the most exciting theorems connecting geometry and physics. The theorem can be understood as a comparison theorem of scalar curvature. It was first proved by Schoen-Yau in 1979, and later by Witten in 1981. They are proofs are so different and even nowadays people do not know how to explain the differences and unify these two proofs.

In this talk, I will explain the basic concepts of scalar curvature and the positive mass theorem. Then I will briefly sketch the two different proofs.

The Hilbert Schemes of points on C^2

Yu Zhao

This talk will cover the definition of Hilbert schemes of points on C^2, the construction as a quotient by group actions, the difference from the symmetric product of a surface, and a discussion of different ways of studying the cohomology groups.

Combinatorial applications of the Hard-Lefschetz Theorem and Hodge-Riemann relations

Christian Gaetz

In the 1980's Richard Stanley used the Hard-Lefschetz theorem from algebraic geometry to solve two famous open problems in combinatorics: the Erdos-Moser conjecture about subset sums and McMullen's g-conjecture for simplicial polytopes. Recently, Adiprasito, Huh, and Katz strengthened these methods by developing "Hodge theory for matroids" and resolved several longstanding conjectures in matroid theory, graph theory, and network theory. The ideas behind these methods and their implications will be discussed.

Graph property testing

Jonathan Tidor

Suppose you are given a very large graph–so large that even reading all of it would be impractical. Is it possible to determine if the graph satisfies some property by only sampling a small subset of the graph?

In this talk we will discuss exactly what this problem means and look at it in a couple different cases: containing a cycle of length four, containing a triangle, and being an expander. Along the way we will discuss connections to other parts of combinatorics.

Wild representation type and how to deal with it

Chris Ryba

When a representation theorist encounters a new algebra, their Pavlovian response is to ask about the module category of the algebra. Perhaps the most obvious question is to ask for a classification of indecomposable modules: this pins down what objects in the category can look like. We will discuss the concept of wild representation type, which can be vaguely thought of as an analogoue of NP-hardness for module categories. This explains why it's often unreasonable to hope for a classification of indecomposable modules. However, we'll also discuss ways in which we can work around this limitation.

(Non-)immersions of real projective spaces

Andy Senger

Given a smooth manifold M, it is a classical problem in differential topology to find the smallest n such that M immerses in $\mathbb{R}^n$. In fact, the celebrated Hirsch-Smale theorem reduces this problem to a purely homotopical one. In this talk, I will explore some of the ways in which methods from homotopy theory can be used to prove (non-)immersion results, focusing on the concrete case of real projective spaces.

About Falconer distance problem in the plane

Hong Wang

If E is a compact set of Hausdorff dimension greater than $5/4$ on the plane, we prove that there is a point $x \in E$ such that the set of distances between x and E has positive Lebesgue measure. Our result improves upon Wolff's theorem for $\dim E > 4/3$. This is joint work with Larry Guth, Alex Iosevich and Yumeng Ou.

Topological Quantum Field Theories and the Cobordism Hypothesis

Robin Elliott

This talk will not be about quantum field theories. It will however be about topological quantum field theories (TQFTs): these are functors from the cobordism category into the category of complex vectors spaces. That is, a TQFT is a rule that assigns a complex vector space to each manifold and assigns linear maps between these vector spaces to cobordisms between these manifolds.

The Cobordism Hypothesis is a classification of all TQFTs, modulo some additional assumptions. This talk will walk through low dimensional examples to get to a statement of the Cobordism Hypothesis. Along the way, I'll highlight the other ingredients in the story: Morse theory, dualisability and a glimpse at higher category theory.

Nodal sets and domains of Laplace eigenfunctions

Paul Gallagher

I'll give a brief introduction to theorems and conjectures on eigenfunctions of the Laplacian on Riemannian manifolds ending with Logunov's proof of Yau's conjecture. Along the way, we'll encounter the frequency function and the saltiest man in geometric analysis.

Picard–Lefschetz Theory

Tim Large

Hamiltonian diffeomorphism groups

Sahana Vasudevan

In this talk I will introduce Hamiltonian diffeomorphism groups from algebraic and geometric perspectives. These are topological groups associated to symplectic manifolds, that also carry a Finsler metric structure called the Hofer metric. On the algebraic side, Hamiltonian diffeomorphism groups are simple groups, but completely determine the underlying symplectic manifold. On the geometric side, the fact that the Hofer metric is actually nondegenerate is a deep result in symplectic geometry. I will talk about these results and related questions.