PHYSICAL MATH SEMINAR

Nonlinear Waves Far from Equilibrium in Hydrodynamic-like Media: Dispersive Shocks and Rogue Waves

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ABSTRACT:

In conservative media, the dispersive regularization of gradient catastrophe gives rise to dispersive shock waves (DSWs). Unlike classical viscous shocks, a DSW is a highly oscillatory nonlinear wavetrain whose leading edge propagates faster than the long-wave speed, while the entire structure expands over time. A powerful framework for describing DSWs is Whitham modulation theory (WMT), a nonlinear WKB-type approach that captures the slow evolution of wave parameters such as amplitude, wavelength, and frequency. DSWs appear in diverse settings, from fluid dynamics and geophysics to nonlinear optics and condensed matter. Rogue waves (RWs), in contrast, are extreme, localized events whose amplitudes far exceed the surrounding background, arising primarily from nonlinear focusing in modulationally unstable media. Both DSWs and RWs are fascinating prototypes of far-from-equilibrium phenomena.

In the first part of my talk, I will present recent work on DSWs in the one-dimensional discrete, defocusing nonlinear Schrödinger equation (DNLS). We focus on strongly discrete regimes approaching the anti-continuum limit (ACL), as well as intermediate regimes bridging the ACL and the continuum limit. We analyze the long time evolution of step initial data with WMT and asymptotic reductions. In particular, this reveals how lattice-induced dispersion modifies shock structure. We identify a sharp discretization threshold beyond which continuum dynamics emerge, while also uncovering a rich variety of intermediate shock morphologies. These results are applied to study shock wave formation in ultracold atomic gases confined in optical lattices.

In the second part, I will describe DSWs and RWs in quantum droplets, an ultracold state stabilized by competing mean-field and beyond mean-field quantum fluctuations. These droplets support diverse shock structures, which we classify using WMT. Prominent among these are dam-break flows (DBFs), which exist in the quantum-fluctuation dominant regime where homogeneous backgrounds are modulationally unstable. We demonstrate controlled generation of rogue waves via head-on interference of DBFs and conclude with ongoing studies in higher-dimensional droplet systems.

TUESDAY, FEBRUARY 3, 2026

2:30 PM – 3:30 PM

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