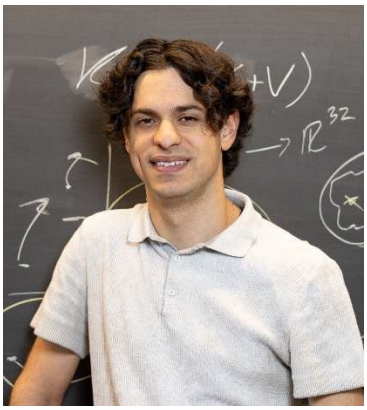
PHYSICAL MATH SEMINAR

A Spectral Theory of Scalar Volterra Equations



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ABSTRACT:

Volterra integral equations have been the subject of much study, from the perspective of both pure mathematics and applied science. Developments in analysis have yielded far-ranging existence, uniqueness, and spectral results for such equations. In applied science, Volterra equations are natural models for signal convolution or filtering, and they describe the evolution of a variety of partially-observed, time-dependent systems. In particular, the study of strain-stress dynamics in materials science has inspired closed-form solutions to special classes of Volterra equations with exponentially decaying memory, as well as certain Volterra equations involving fractional derivatives and Prony series. Only a limited number of these results have been proven rigorously, however, and their study in materials science has remained largely disjoint from the broader mathematical community.

In this talk, we develop a spectral theory for scalar, linear Volterra equations, showing that a variety of disparate results in applied science (including the aforementioned results in viscoelasticity) are special cases of a more general theory. In particular, we derive analytic solutions for large classes of continuous or discrete-time linear Volterra equations, as well as fractional and delay differential equations.

We show how our closed-form solutions can be realized numerically with rational approximation, study the analytical properties of these formulas, and test them on a wide array of problems relating to signal deconvolution, triangular matrix inversion, and interconversion of materials.

TUESDAY, MARCH 11, 2025

2:30 PM – 3:30 PM

Building 2, Room 449

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