PHYSICAL MATH SEMINAR

Topological Defects, Topological Barriers, and Convex Relaxation

David Palmer

Harvard

ABSTRACT:

Geometric optimization problems are full of topological barriers that get in the way of local optimization. Convex relaxation is a powerful guide and tool for reframing such problems.

In optimization over surfaces, local methods can get stuck when the incorrect topology is chosen at initialization. Current relaxation, an idea recently borrowed from geometric measure theory into computational practice, provides an alternative convex language for surface optimization that avoids these barriers. We combine current relaxation with a neural implicit representation to devise a general optimization approach for learning families of surfaces with boundary.

Next we turn to hexahedral meshing, an essential geometric prerequisite to many techniques for simulating continuous physical systems. In turn, the seemingly abstract computational structure of a hexahedral mesh features topological defects analogous to those found in physics. Moreover, the topological barriers these defects present are at the core of what makes computing hex meshes so challenging. Through exploring the geometry of defects and associated octahedral frame fields, we devise two convex relaxation approaches to surmounting these barriers. In the first approach, an SDP relaxation of projection onto the space of frames enables better escaping local minima. In the second approach, lifting optimization from fields to their graphs offers a new, more geometric perspective on defects and simultaneously suggests a convex current relaxation.

Finally, I will discuss some open questions and ongoing work. Field-based meshing can fail catastrophically when a computed defect configuration is not realizable in a mesh, reflecting a murky distinction between field and mesh defect topology. A better understanding of global defect configurations would also unlock more efficient optimization approaches through reduced-order modeling. I will suggest how the tools of lifting and convex relaxation might aid in this problem, as well as in other geometric optimization problems for applications in medical imaging and computational engineering.

TUESDAY, SEPTEMBER 17, 2024

2:30 PM – 3:30 PM

Building 2, Room 449