Physical Mathematics Seminar

Solving PDEs exactly over polynomials

KEATON BURNS

Massachusetts Institute of Technology

ABSTRACT:

Numerical simulations of partial differential equations (PDEs) are an indispensable tool across science and engineering. For simple geometries, spectral methods are a powerful class of techniques that produce extremely accurate solutions for broad ranges of equations. But many flavors of these methods exist with different properties and performance, and developing the best methods for complex nonlinear problems is often quite challenging.

Here we present a framework that unifies all polynomial and trigonometric spectral methods, from classical "collocation" to more recent "ultraspherical" schemes. In particular, we examine the exact equations satisfied by each method's discrete solution, which differ from the original PDE by perturbations called "tau corrections". By analyzing these corrections, we can precisely categorize existing methods and derive new solvers that robustly implement new boundary conditions, eliminate spurious numerical modes, and satisfy exact conservation laws.

This approach conceptually separates *what* discrete model a spectral scheme solves from *how* it solves it, allowing for much more freedom when developing new solvers and analyzing their solutions. Along the way, we will illustrate these advantages with examples from the Dedalus Project, an open-source package for solving PDEs with modern spectral methods.

TUESDAY, MARCH 7, 2023
2:30 PM – 3:30 PM
Building 2, Room 449

https://math.mit.edu/sites/pms/

Massachusetts Institute of Technology