PHYSICAL MATHEMATICS SEMINAR

Isometric immersions and self-similar buckling in Non-Euclidean elastic sheets

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ABSTRACT:

The rippling patterns observed in torn elastic sheets, leaves and swelling hydrogels provide striking examples of periodic and self-similar patterns. Within the formalism of finite elasticity, it is understood that such patterns arise from the sheet buckling to relieve growth induced residual strains. While numerical experiments set within this framework have been able to replicate these patterns, there is no complete theoretical understanding of the mechanism behind the self- similar patterns. On the one hand, such complex, self-similar patterns are usually associated with "strongly frustrated" systems, e.g. elastic sheets with boundary conditions that preclude the possibility of relieving in plane strains, or at alloy-alloy interfaces between distinct crystal structures. On the other hand, many growth patterns generate residual in-plane strains which can be entirely relieved by the sheet forming part of a surface of revolution or a helix. Given that generically this system is not strongly frustrated, why then do we observe self-similar buckling patterns? In this talk we address this puzzle by showing that for a large class of growth profiles there exist periodic and self-similar deformations of the sheet with vanishing in-plane strain. The construction of these surfaces consists of gluing together local solutions of an isometric immersion problem along "lines of inflection" and "branch points" in such a manner that the resulting surface has finite bending energy. We propose that the sheet introduces these defects to locally reduce large bending content which necessarily results from the extrinsic geometry of the isometric immersions. For hyperbolic non-Euclidean sheets, complex wrinkling patterns are thus possible and our results identify the key role the regularity of the isometric immersion plays in determining the global structure of a non-Euclidean elastic sheet.

TUESDAY, SEPTEMBER 15, 2015
2:30 PM
Building E18, Room 466A

Reception following in Building E17, Room 401A
(Math Dept. Common Room)

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