

THE MATHEMATICAL THEORY OF WAVE TURBULENCE

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Wave turbulence is the theory of nonequilibrium statistical mechanics for wave systems. Initially formulated in pioneering works of Peierls, Hasselman, and Zakharov early in the past century, wave turbulence is widely used across several areas of physics to describe the statistical behavior of various interacting wave systems. We shall be interested in the mathematical foundation of this theory, which for the longest time had not been established.

The central objects in this theory are: the "wave kinetic equation" (WKE), which stands as the wave analog of Boltzmann's kinetic equation for interacting particle systems, and the "propagation of chaos" hypothesis, which is a fundamental postulate in the field that lacks mathematical justification. Mathematically, the aim is to provide a rigorous justification and derivation of those two central objects; This is Hilbert's Sixth Problem for waves. In this talk, we shall survey some recent progress on this problem that happened over the past few years, leading up to our work with Yu Deng (University of Southern California) in which we give a rigorous derivation of the wave kinetic equation and a justification of the propagation of chaos, in the context of the nonlinear Schrodinger equation.