COUNTING CLOSED GEODESICS AND IMPROVING WEYL'S LAW FOR PREDOMINANT SETS OF METRICS

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We discuss the typical behavior of two important quantities on compact manifolds with a Riemannian metric g: the number, $c(T, g)$, of primitive closed geodesics of length smaller than T, and the error, $E(L, g)$, in the Weyl law for counting the number of Laplace eigenvalues that are smaller than L. For Baire generic metrics, the qualitative behavior of both of these quantities has been understood since the 1970's and 1980's. In terms of quantitative behavior, the only available result is due to Contreras and it says that an exponential lower bound on $c(T, g)$ holds for g in a Baire-generic set. Until now, no upper bounds on $c(T, g)$ or quantitative improvements on $E(L, g)$ were known to hold for most metrics, not even for a dense set of metrics. In this talk, we will introduce the concept of predominance in the space of Riemannian metrics. This is a notion that is analogous to having full Lebesgue measure in finite dimensions, and which, in particular, implies density. We will then give stretched exponential upper bounds for $c(T, g)$ and logarithmic improvements for $E(L, g)$ that hold for a predominant set of metrics. This is based on joint work with J. Galkowski.