MICROLOCAL ANALYSIS OF INTERNAL WAVES IN 2D AQUARIA

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For a bounded smooth planar domain \( \Omega \), we study the forced evolution problem for the 4th order PDE

\[
(\partial_t^2 \Delta + \partial_{xx}^2) u(t, x) = f(x) \cos(\lambda t), \quad t \geq 0, \quad x \in \Omega
\]

with homogeneous initial conditions and Dirichlet boundary conditions on \( \partial \Omega \). This is motivated by concentration of fluid velocity on attractors for stratified fluids in effectively 2-dimensional aquaria, first observed experimentally in 1997.

The behavior of solutions to (1) is intimately tied to the chess billiard map on the boundary \( \partial \Omega \), which depends on the forcing frequency \( \lambda \). Under the natural assumption that the chess billiard \( b \) has the Morse–Smale property, we show that as \( t \to \infty \) the singular part of the solution \( u \) concentrates on the attractive cycle of \( b \). The proof combines various tools from microlocal analysis, scattering theory, and hyperbolic dynamics. Joint work with Jian Wang and Maciej Zworski.