RUELLE ZETA AT ZERO FOR NEARLY HYPERBOLIC 3-MANIFOLDS

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For a compact negatively curved Riemannian manifold (Σ, g), the Ruelle zeta function $\zeta_{\mathrm{R}}(\lambda)$ of its geodesic flow is defined for $\operatorname{Re}\lambda \gg 1$ as a convergent product over the periods T_γ of primitive closed geodesics

$$\zeta_{\mathrm{R}}(\lambda) = \prod_\gamma (1 - e^{-\lambda T_\gamma})$$

and extends meromorphically to the entire complex plane. If Σ is hyperbolic (i.e. has sectional curvature -1), then the order of vanishing $m_{\mathrm{R}}(0)$ of ζ_{R} at $\lambda = 0$ can be expressed in terms of the Betti numbers $b_j(\Sigma)$. In particular, Fried proved in 1986 that when Σ is a hyperbolic 3-manifold,

$$m_{\mathrm{R}}(0) = 4 - 2b_1(\Sigma).$$

I will present a recent result joint with Mihajlo Cekić, Benjamin Küster, and Gabriel Paternain: when $\dim\Sigma = 3$ and g is a generic perturbation of the hyperbolic metric, the order of vanishing of the Ruelle zeta function jumps, more precisely

$$m_{\mathrm{R}}(0) = 4 - b_1(\Sigma).$$

This is in contrast with dimension 2 where $m_{\mathrm{R}}(0) = b_1(\Sigma) - 2$ for all negatively curved metrics. The proof uses the microlocal approach of expressing $m_{\mathrm{R}}(0)$ as an alternating sum of the dimensions of the spaces of generalized resonant Pollicott–Ruelle currents and obtains a detailed picture of these spaces both in the hyperbolic case and for its perturbations.