WEIL-PETERSSON CURVES, BETA-NUMBERS AND MINIMAL SURFACES

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Weil-Petersson curves are the closure of the smooth planar closed curves for the Weil-Petersson metric on universal Teichmuller space defined in 2006 by Takhtajan and Teo. Their work was motivated by problems in string theory, but the same class of curves arises naturally in geometric function theory, computer vision, and the theory of Schramm-Loewner evolutions (SLE). Giving a geometric characterization of these curves had been a open problem, but I shall describe several such characterizations in terms of quantities such as Sobolev smoothness, Mobius energy, fixed curves of biLipschitz involutions, Peter Jones's beta-numbers, the thickness of hyperbolic convex hulls, the total curvature of minimal surfaces in hyperbolic space, and the renormalized area of these surfaces. The last of these results gives another connection to physics and generalizes a result of Alexakis and Mazzeo. Moreover, most of these characterizations extend to higher dimensions and remain equivalent there, defining new classes of curves that may be of interest.