VORTEX FILAMENT SOLUTIONS OF THE 3D NAVIER-STOKES SOLUTIONS

JACOB BEDROSSIAN

We consider solutions of the Navier-Stokes equations in 3d with vortex filament initial data of arbitrary circulation, that is, initial vorticity given by a divergence-free, vector-valued measure of arbitrary mass supported on a smooth curve. First, we prove global well-posedness for perturbations of the Oseen vortex column in scaling-critical spaces. Second, we prove local well-posedness (in a sense to be made precise) when the filament is a smooth, closed, non-self-intersecting curve. Besides their physical interest, these results are the first to give well-posedness in a neighborhood of large self-similar solutions of 3d Navier-Stokes, as well as solutions which are locally approximately self-similar. In particular, in velocity form, the initial condition is large in BMO^{-1}, a critical space in which local well-posedness of large data is unknown and conjectured false (but is not in the usual smaller critical spaces, such as weak L^3).