ON THE LOEWNER ENERGY OF JORDAN CURVES AND WEIL–PETERSSON QUASICIRCLES

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We introduce the Loewner energy for Jordan curves and relate this quantity to ideas and concepts coming from random conformal geometry, geometric function theory, and Teichmuller theory. Many of those concepts arise from mathematical physics.

One motivation for the definition of the Loewner energy is that it describes the large deviations of a family of simple random curves called Schramm-Loewner evolutions (SLE) of vanishing parameter. This provides a probabilistic interpretation of the Loewner energy that allows us to establish several interesting invariance properties. We further derive an equivalent characterization of the Loewner energy using zeta-regularized determinants of Laplacians. This then identifies the family of finite energy curves to the family of so-called Weil–Petersson quasicircles. Moreover, the Loewner energy is shown to equal the Kähler potential, introduced by Takhtajan and Teo, of the unique Kähler metric on the corresponding Weil–Petersson Teichmuller space. If time allows, I will also present the conjectural connection to minimal surfaces in the hyperbolic 3-space.